Term Limits and Bargaining Power in Electoral Competition

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Abstract

I study a dynamic model of electoral competition between candidates with heterogeneous valence. When the candidates' and voters' policy preferences differ, the winner extracts rents, limited only by the voters' threat of electing the weaker candidate. This threat becomes more costly to the voters when the relevant time horizon is longer. Thus, term limits can increase the voters' bargaining power and their welfare. Term limits are even more important for curbing rent extraction if entry is strategic, as in that case strong incumbents face weaker competition. The paper also compares the welfare properties of seniority caps and stochastic term limits.

1 Introduction

The design of presidential term limits has garnered much debate. Most democracies stipulate some form of term limits, restricting the number of times that a person can be elected president. For example, in the United States, a politician can serve as president for at most two four-year terms in total. This is a common system, but many countries have a limit of one or three terms instead. Term limits may also apply to other elected officials, such as governors or legislators. The tradition of term limits

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goes back to ancient Greece and the Roman Republic, where many public offices had such constraints: for example, Roman consuls served for one year at a time and had to wait several years to stand for election again.

Proponents of term limits argue that such constraints are a useful check on executive power. As the argument goes, frequent turnover of politicians is necessary to maintain a healthy democracy; removing term limits would lead to a powerful president becoming entrenched, with detrimental consequences. This fear is present even in well-established republics where abuse of formal power or outright dictatorship are unlikely outcomes. For example, prior to the Second World War, the United States had an informal rule by which presidents did not seek to run for more than two terms. Franklin D. Roosevelt broke this rule by successfully running four times, although he died a year after being elected in 1944 for the fourth time. Seeing this as a threat to democracy, Congress subsequently passed the Twenty-second Amendment, formalizing the two-term limit. In the words of 1944 Republican nominee Thomas Dewey, who campaigned in favor of the reform, "Four terms, or sixteen years, is the most dangerous threat to our freedom ever proposed" (Jordan, 2011).

However, the welfare impact of term limits in a standard model of electoral competition is unclear. In a model where potential candidates and platforms are fixed (or determined independently of term limits), the only effect of term limits is to restrict the choice set faced by voters, leaving them worse off. Introducing an *ex post* effort decision may tilt the scales further against term limits, since the prospect of reelection can incentivize effort.[1]

What are term limits good for, then? Are they necessary in strong democracies? Or are they only justified in weak democracies, where a shift to non-democracy or institutional decay are real dangers?

To answer these questions, we study a dynamic model of elections with complete information in which candidates compete by choosing policies but also differ in their ability, and challengers may strategically choose whether to contest an election. We assume that, with some frequency, candidates are biased and seek to choose policies not preferred by the voters—their ability to do so is curbed only by the threat of losing to the opposition. However, there is no formal channel for an executive overreach or

[1] For instance, in Ferejohn (1986), politicians are homogeneous but choose an effort level while in office, and voters use a retrospective rule to reward effort. Reed (1994) extends the model to the case where politicians have heterogeneous ability. In both cases, term limits would remove incentives and make such rules less effective.

democratic backsliding.

The paper makes two contributions. First, even in the absence of term limits, the paper develops a novel framework for studying dynamic electoral competition. Second, the paper identifies the forces that make term limits beneficial or detrimental within this model.

We first consider a benchmark case in which challengers are non-strategic and always enter when the opportunity presents itself. We show that, when politicians are usually biased (i.e., their policy preferences differ from the voters'), a one-term limit is optimal, while when they are usually unbiased, having no term limits is optimal. That term limits may be desirable even when entry is non-strategic (so that there is no scope for entry deterrence) may be surprising. The logic behind the result is that term limits do two things. First, they sometimes force voters to inefficiently discard good incumbents. Second, by limiting the maximum tenure of any incumbent, term limits alleviate the dynamic concerns voters face when choosing between candidates. Indeed, with a one-term limit, the continuation is independent of who is elected today; without term limits, a bad incumbent can stay in the system longer if elected, generating a bad continuation as well. Weak term limits thus lead to bigger gaps between the attractiveness of good and bad candidates, and it is these gaps that the winner of an election leverages to choose her preferred policies with impunity when she is biased.

Optimal term limits must solve this trade-off between optimal retention of talent and limiting the bargaining power of strong candidates. Our first main result is that, when politicians are often biased, the latter effect is strong enough to warrant term limits.

The full model with strategic challenger entry introduces two additional concerns to the analysis. On the one hand, as might be expected, challengers are less likely to run against a strong incumbent; the danger that a good candidate will be able to exploit the voters is thus more severe, and term limits more critical. On the other hand, if entry is relatively costly, imposing term limits may lower the candidates' expected gains from running for office to the point that voters are not offered any high-ability candidates at all.

Our approach to modeling term limits is flexible. In general, we allow for stochastic term limits which allow an incumbent in their kth period to run for reelection with probability p_k. This nests conventional term limits (or as we will call them, *classic* term limits) as a special case. Many of our results concern *stationary* term limits, which is to say limits in which p_k is not a function of k. This class of term limits turns out to be uniquely tractable and, in some cases, welfare-superior to classic term limits. Although uncommon in practice, it does contain two important special cases: $p_k \equiv 1$ corresponds to no term limits, while $p_k \equiv 0$ gives a one-term limit.

Our analysis also shows that non-stationary term limits (including, in particular, classic term limits) create artificial incentives to sometimes elect the lower-ability candidate, due to differences in seniority or the expected future impact of a candidate on competition. In particular, classic term limits can create an *incumbency advantage*, incentivizing voters to reelect the incumbent *over an equally qualified challenger*.[2] The logic is that open elections are desirable by virtue of generating more competition. Then, since classic term limits are more binding for senior politicians, voters know that the fastest way to get a new open election is to reelect the incumbent until her time runs out; electing a challenger would reset the clock.

Finally, in an extension we discuss flexible term limits, i.e., a setting in which voters can choose on the fly to extend the incumbent's clock. This is an important case since many attempts to change term limits are led by an incumbent, rather than made *ex ante* by a social planner. Here voters discard bad incumbents and allow good incumbents to run for reelection, but when the cost of entry is high (i.e., there is a high potential for deterrence), they also deny very good incumbents, a form of "fear of tyranny".

Our paper ties into the theoretical literature on dynamic elections and term limits. The way we model electoral competition is related to Ashworth and Bueno de Mesquita (2008, 2009). The former presents a two-period model with strategic entry but fixed platforms, while the latter studies a static model with strategic valence investments and platform choice. Although both share ingredients with this paper, it is the combination of all three elements—strategic entry and platform choice in a fully dynamic setting—that underpins our results.

[2]This effect is different from other mechanisms proposed in the literature, such as the *electoral selection* effect (Ashworth and Bueno De Mesquita, 2008; Gowrisankaran, Mitchell and Moro, 2008; Zaller, 1998), whereby incumbents are better in expectation than the average candidate by virtue of having won past elections; and the *strategic challenger entry* effect (Ashworth and Bueno De Mesquita, 2008; Gordon, Huber and Landa, 2007b; Cox and Katz, 1996; Stone, Maisel and Maestas, 2004), whereby challengers are deterred from running against good incumbents. These effects—which are also encompassed by our model—would also lead to incumbents being reelected with high probability, but not conditional on the challenger being of equal ability.

There is a large literature on electoral signaling, i.e., models in which the incumbent's ability (Rogoff and Sibert, 1988; Rogoff, 1990) or preferences (Alesina and Cukierman, 1990; Hess, 1991; Smart and Sturm, 2013) are private information, and policies or economic performance serve as a signal. Aghion and Jackson (2014) studies a contracting problem in which a principal wants to incentivize an agent to take a risky action that may reveal her type. In Harrington Jr (1993), the effectiveness of different policies is uncertain and electoral pressures may drive politicians to pander to the voters' beliefs. The general point of these models is that reelection prospects lead to signaling behavior, which may be good (e.g., when it induces effort) or bad (e.g., when it induces budget cycles or inaction). Term limits tend to mute incentives to signal.

Some papers in this literature deal specifically with term limits. In Bernhardt, Dubey and Hughson (2004), the voters reelect the incumbent only if her policy is moderate enough; with term limits, voters subject senior incumbents to more stringent "ideological tests" to prevent an extreme type from reaching her last possible term, in which she will be fully unconstrained. Banks and Sundaram (1998) and Duggan (2017) study infinite-horizon models in which agents can only be retained for two periods (that is, a two-term limit is assumed), and there is moral hazard and adverse selection. Electoral concerns can incentivize agents to exert more effort in their first period, but not too much—otherwise voters would be too tempted to get a new challenger in every period and electoral incentives would vanish. Another strand of this literature (Ashworth, 2005) uses the career concerns framework, in which symmetric uncertainty about the politician's ability drives her to exert effort.

Our model departs from this literature by assuming that abilities are common knowledge (hence no signaling), and that policy platforms are chosen *before* each election, as in the Hotelling-Downs model. Another paper in the spirit of dynamic Hotelling-Downs competition is Forand (2014): there, two parties compete by offering policies in every period, except that the incumbent's position is fixed until she loses. This is similar to how, in this paper, the incumbent's party does not draw new challengers—hence cannot change its ability. In Forand (2014), policy choices are tricky because politicians want to increase their policy payoffs but not choose policies that can be easily attacked in the future. This concern does not arise in our model because the sticky variable is ability, which is drawn randomly, not chosen.

There is also a sizable empirical literature on term limits, policy choice and the incumbency advantage, which is broadly in line with the mechanics and predictions of this paper. For instance, Gowrisankaran et al. (2008) argue that incumbents with long tenure in the U.S. Senate deter challenger entry, and term limits would increase welfare by preventing this. Ban, Llaudet and Snyder (2016) estimate that 30% to 40% of the incumbency advantage in U.S. state legislatures is the result of deterring experienced challengers, and Carey, Niemi and Powell (2000) and Lee (2008) provide additional evidence of the incumbency advantage for U.S. Representatives and its causes. These papers support our assumption of *strategic challenger entry* and its logical consequence, entry deterrence. On the other hand, Gordon et al. (2007a) show that judges facing electoral competition cater to voters by being harsher; and Acemoglu, Reed and Robinson (2013) find that chiefs in Sierra Leone facing less competition generate worse economic outcomes. This parallels our prediction that politicians take advantage when unconstrained by challengers. Our model predicts that (classic) term limits may strengthen the incumbency advantage. This prediction is consistent with Querubín (2011), who finds that the introduction of term limits to the Philippines made incumbents safer prior to the end of their tenure.

The rest of the paper proceeds as follows. Section 2 presents the baseline model with exogenous entry. Section 3 characterizes the equilibrium and its welfare properties. Section 4 adds strategic entry to the model. Section 5 considers the case of flexible term limits. Section 6 concludes. All proofs are in Appendices A and B.

2 The Model

Time $t = 0, 1, 2, \ldots$ is discrete and infinite. There are two types of players: a representative voter with ideal policy $x = 0$,[3] and a set of politicians who enter and leave the model according to the rules of the electoral process.

In each period a politician is elected president. The voter cares both about the incumbent's ability and her policy platform. The voter's utility function at time t is:

$$U_v^t\left((\theta_s)_{s\geq t}, (x_s)_{s\geq t}\right) = \sum_{s=t}^{\infty} \delta^{s-t} u_v(\theta_s, x_s) = \sum_{s=t}^{\infty} \delta^{s-t}\left(\theta_s - \lambda x_s^2\right), \tag{1}$$

where θ_s is the ability of the incumbent at time s; x_s is the policy at time s; and

[3]The results extend to a setting with a continuum of voters, with the median voter taking the role of the representative voter.

$\lambda > 0$ is a parameter that reflects the relative importance of valence *vs.* ideology.[4] Politicians are chosen as follows. There are two parties, L and R. Every period, each party presents a candidate to the election. If the incumbent can run for reelection, she automatically becomes her party's candidate and there is a *closed election*. Otherwise there is an *open election*.

Parties without eligible incumbents produce challengers. To simplify matters we assume that politicians who have run in the past and lost, or become term-limited, cannot return to politics, so challengers are always fresh draws from the pool. The ability θ of a prospective challenger is drawn from a distribution F, given by a bounded, continuous density f with support $[0, 1]$. A candidate's ability is permanently fixed once drawn.

The utility of a politician i of party P at time t is given by

$$W_i^t((I_s)_{s\geq t}, (x_s)_{s\geq t}) = \sum_{s=t}^{\infty} \delta^{s-t} \left[b - \gamma(\alpha_{is} - x_s)^2\right] \mathbb{1}_{is}, \tag{2}$$

where $\mathbb{1}_{is}$ equals 1 if i is in power at time s and 0 if not; b represents baseline rents from office; γ reflects the relative importance of policy preferences *vs.* holding office; and α_{is} is i's preferred policy, or bliss point, in period s. (Note that the politician is assumed to only care about policy when he is in office.)

We model politicians' bliss points as follows. In each period t, a politician has probability μ of being ideologically *biased* relative to the voter, and probability $1 - \mu$ of being *unbiased*. These are iid draws across periods and politicians. An unbiased politician has bliss point 0 in that period; an biased politician of party R has bliss point $I > 0$ in that period; and a biased politician of party L has bliss point $-I < 0$ in that period.

As a motivating story for this assumption, suppose each election is about a different "main issue" and platforms revolve around them; a generally biased politician may be highly biased on some issues but not others. The constant μ parameterizes the degree of conflict of interest between voters and politicians.[5]

[4] Alternatively, $x_s \geq 0$ can represent rent-seeking or corrupt behavior by the politician. In this case the politicians want as much corruption as possible. Most results below can be reformulated accordingly.

[5] An apparently more straightforward assumption would be that politicians are always biased by the same amount, but I may be large or small. Our approach is qualitatively similar but yields more tractable results.

We will make the following two parameter assumptions. First, I is large enough that a biased politician's bliss point is never a viable platform: formally, $I \geq \sqrt{\frac{1}{(1-\delta)\lambda}}$. Second, b is high enough that a politician's flow payoff while in office is always positive: $b > \gamma I^2$.

Electoral competition takes place as follows. Once each party has produced a candidate, their abilities and bias states for that period are publicly revealed; then each candidate i simultaneously picks a policy platform $x_{it} \in \mathbb{R}$ that will be implemented if i wins. (This is a credible commitment, but no promises can be made about what platforms will be offered in future elections.)[6] Given these observed abilities and platforms, the voter elects the politician offering her the highest expected utility.

2.1 Term Limits

In general we allow for stochastic and seniority-dependent term limits. Formally, a term limit will be given by a sequence $(p_k)_{k\geq 1}$ of probabilities, where p_k is the probability that an incumbent in her k-th term is allowed to run again.[7]

We will define three special classes of term limits. A term limit $(p_k)_{k\geq 1}$ has *finite horizon* if $p_m = 0$ for some m. For $m \in \mathbb{N}$, a *classic* m-term limit is given by $p_k = 1$ for $k < m$ and $p_m = 0$. This is what is conventionally called an m-term limit. A *stationary* p-term limit is one in which the incumbent is allowed to run again with probability p in every history, i.e., $p_k = p$ for all k.

There are three reasons for studying general—in particular, stochastic—term limits. First, stationary term limits, in particular, are very tractable, while illustrating the same forces at work under classic term limits. Second stochastic term limits can yield superior welfare outcomes; they are thus an institutional innovation worth considering. Third, they can be seen as modeling the randomness of weakly institutionalized democracies.[8]

[6] A concern regarding term limits is that term-limited incumbents may behave differently in office since they lack electoral incentives. As shown by the incomplete information literature, this may be good or bad for voters. Since this is not the focus of our model, a reasonable choice is to set the effect of this channel to zero; our assumption that politicians have the same ability to commit to platforms regardless of seniority accomplishes this.

[7] Even more generally, we could consider mappings $p : H \longrightarrow [0,1]$ with the set of histories of the game as the domain. The incumbent's probability of being able to run again could then depend on her ability, the ability and tenure length of past incumbents, etc. In practice, however, ability is hard to measure objectively and may hence be non-contractible. We do present an extension with some of this flavor in Section 5.

[8] A stationary p-term limit can represent a society with no term limits in which the incumbent

2.2 Timing

The structure of the game is as follows. In each period t,

1. Nature decides (and reveals) if the incumbent elected at the end of period $t-1$ can run again in period t.

2. Parties without an incumbent draw challengers from the pool; their abilities are realized and publicly revealed.

3. The candidates' biases in period t are drawn and publicly revealed.

4. Candidates simultaneously make campaign promises for period t.

5. The voter chooses the winner and flow payoffs are realized.

2.3 Definition of Equilibrium

Our solution concept will be Markov Perfect Equilibrium (MPE). That is, actions are assumed to depend only on the payoff-relevant components of the history: policy choices depend only on the candidates' abilities and seniority, while votes depend on abilities, seniority and platforms.[9][10] In particular, the restriction to Markovian equilibria rules out punishment strategies by the voter that could hypothetically force politicians to offer more centrist platforms.

We will further focus on *symmetric* MPE (SMPE). This means that strategies and value functions will be equal for candidates of both parties. Finally, under stationary term limits, we will restrict attention to *stationary* symmetric MPE (SSMPE), meaning that strategies and value functions will also be independent of the incumbent's seniority. In general we allow for mixed strategies; for simplicity, we will restrict our notation to pure strategies when doing so entails no loss of generality.[11]

has a probability $1-p$ of losing her chance to run due to a scandal, criminal charges, etc., or a society with a one-term limit in which the rules can be flouted with probability p.

[9]While seniority does not affect current payoffs, it affects the incumbent's prospect of reelection, so it is payoff-relevant.

[10]In particular, our definition of MPE does not allow for conditioning on calendar time. This solution concept is sometimes called *stationary MPE* in the literature. We will instead reserve the word *stationary* for when the MPE also does not condition on seniority.

[11]For example, we will simply talk of candidates choosing policies rather than probability distributions over policies, since all policy choices in all equilibria turn out to be in pure strategies.

2.4 Value Functions

We will use the following notation to denote the voter's equilibrium utility in different states. Holding fixed a Markov strategy profile, V is the voter's (expected) utility at the beginning of the game. This is also the voter's continuation utility from an open election.[12]

$V_k(\theta)$ $(k \geq 1)$ is the voter's utility from a closed election, with an incumbent of ability θ and seniority k. That is, $V_k(\theta)$ is the voter's continuation utility at any history h in which Nature has just revealed that the incumbent (of ability θ, who has been elected k times) will be allowed to run for her $(k+1)$-th term, and the challenger's ability has **not** yet been realized.

$U_k(\theta)$ $(k \geq 0)$ is the voter's utility from electing a candidate of ability θ and seniority k *if that candidate makes her best offer*, $x_t = 0$, in the current election. More generally, if a politician (θ, k) offers platform x, the expected utility she offers to the voter is $U_k(\theta) - \lambda x^2$. We will refer to $U_k(\theta)$ as the politician's *electoral strength*.

Note that, by definition, for all $k \geq 0$

$$U_k(\theta) = \theta + \delta p_{k+1} V_{k+1}(\theta) + \delta(1 - p_{k+1})V. \tag{3}$$

This simplifies under particular schemes. For example, under stationary limits this becomes $U(\theta) = \theta + \delta p V(\theta) + \delta(1-p)V$. Under an m-term classic limit, $U_k(\theta) = \theta + \delta V_{k+1}(\theta)$ for $k = 0, \ldots, m-2$ and $U_{m-1}(\theta) = \theta + \delta V$.

Some of our results will concern the welfare effects of term limits. Our main measure of welfare will be the voter's expected utility, V.

3 Analysis

In this section we characterize the equilibria of the game. We first analyze the equilibrium policy choices by the candidates, and the voter's decisions, taking the value functions $U_k(\theta)$, $V_k(\theta)$ as given. We then characterize the value functions as precisely as possible; an explicit solution is given for stationary term limits. Finally, we characterize the optimal term limits as a function of the parameters of the model.

Let us begin with a couple of observations. In each election, the voter must choose

[12] More precisely, V is the voter's continuation utility at any history h where Nature has just revealed that the incumbent will not be allowed to run again.

the candidate offering her the highest expected utility. By definition, this would be the candidate with higher electoral strength, if both candidates were making their best offers ($x_{it} = x_{jt} = 0$). Our first result is that this is indeed the candidate who wins in equilibrium; but, if she is biased, she will only make the weakest offer needed to beat the losing candidate.

Proposition 1. *In any MPE, in any election at time t between two candidates i, j with respective seniorities k_i, k_j and abilities θ_i, θ_j, suppose that $U_{k_i}(\theta_i) > U_{k_j}(\theta_j)$. Then i wins the election. If i is biased at time t, i offers $|x_{it}| = \sqrt{\frac{U_{k_i}(\theta_i) - U_{k_j}(\theta_j)}{\lambda}}$, with x_{it} positive if i belongs to R and vice versa, and j offers $x_{jt} = 0$ or an equivalent mixed strategy.*[13] *The voter, though indifferent, chooses i; her continuation utility is $U_{k_j}(\theta_j)$. If i is unbiased at time t, i offers $x_{it} = 0$, j's policy choice is indeterminate, and the voter's payoff is $U_{k_i}(\theta_i)$.*

The intuition behind this result is as follows. If the stronger candidate, i is unbiased, her interests align with the voter's, so she offers $x_{it} = 0$ and the voter is strictly better off electing her; there is nothing j can do. If i is biased, on the other hand, and j is offering $x_{jt} = 0$, then i maximizes her payoff by leaving the voter indifferent. Naturally, if j offered any other policy, i would best-respond with a policy even further from 0, but then j could deviate back to 0 to win the election.

The situation when the winner is biased is akin to a bargaining game between the stronger candidate and the voter, in which the candidate makes a take-it-or-leave-it offer—hence has all the bargaining power—and the voter's threat point, her only leverage, is to elect the weaker candidate instead. In this situation, the greater the gap in electoral strengths between the two candidates, the better the winner's bargaining position. This insight underpins the main results of the paper.

We will now characterize the value functions. Because the winner of each election is biased with probability μ, it follows from Proposition 1 that, for all $k \geq 1$,

$$V_k(\theta) = \mu E\left[\min(U_k(\theta), U_0(\theta'))|\theta' \sim F\right] + (1-\mu)E\left[\max(U_k(\theta), U_0(\theta'))|\theta' \sim F\right]. \quad (4)$$

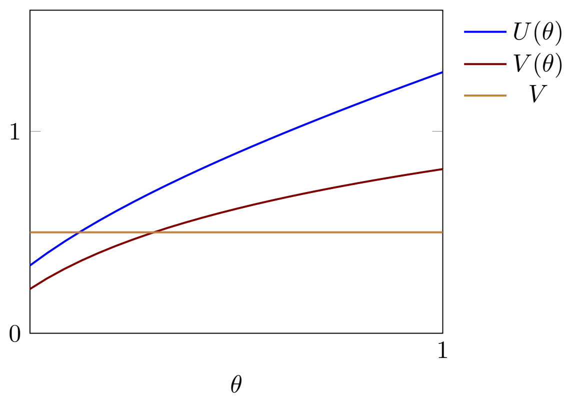
Analogously, the voter's expected utility from an open election is

$$V = \mu E\left[\min(U_0(\theta), U_0(\theta'))|\theta, \theta' \sim F\right] + (1-\mu)E\left[\max(U_0(\theta), U_0(\theta'))|\theta, \theta' \sim F\right]. \quad (5)$$

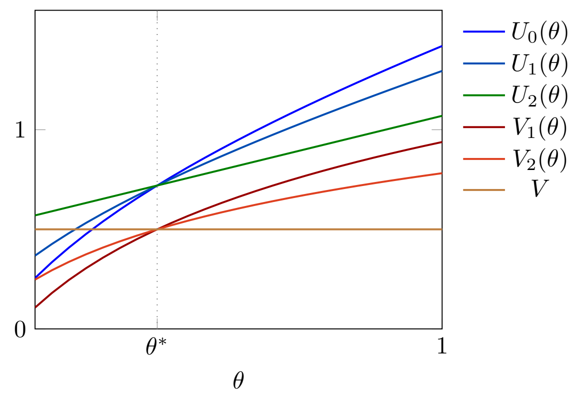
[13] There are mixed-strategy equilibria in which j mixes with high enough weight near 0 that i's best response is unchanged; they are payoff-equivalent to the equilibrium shown.

Equations 3 and 4 for all k, together with Equation 5, allow us to reduce the problem of finding the functions $U_k(\theta)$, $V_k(\theta)$ to a fixed point problem. For instance, suppose that $p_m = 0$ for some m, and conjecture that the function $U_0(\theta)$ equals some candidate function $\hat{U}_0(\theta)$. Under this conjecture, we can calculate a conjectured value of V from Equation 5; U_{m-1} from Equation 3; V_{m-1} from Equation 4; U_{m-2} from Equation 3; V_{m-2} from Equation 4; and so on until we obtain a new conjecture for U_0. This collection of calculated value functions is supported by an equilibrium strategy profile iff the new conjecture of U_0 matches the original conjecture.

Proposition 2 derives some concrete implications of this argument.



(a) Equilibrium under stationary limits



(b) Equilibrium under a 3-term limit

Figure 1: Value functions

Proposition 2. *Assume either stationary or finite horizon term limits. Then:*

(i) There is a unique SMPE.

(ii) If the term limits have finite horizon, for each k, $U_k(\theta)$ and $V_k(\theta)$ are continuous and strictly increasing in θ.

(iii) If the term limits are p-stationary, $U'(\theta) \equiv \frac{1}{1-\delta p[\mu+(1-2\mu)F(\theta)]}$ and $V'(\theta) \equiv \frac{\mu+(1-2\mu)F(\theta)}{1-\delta p[\mu+(1-2\mu)F(\theta)]}$. $U(0)$, $V(0)$ and V can also be calculated explicitly.

(iv) Under m-classic limits, there is $\theta^* \in (0,1)$ such that $U_k(\theta^*)$ is constant in k and $V_k(\theta^*) = V$ for all k. $U_k(\theta)$ and $V_k(\theta)$ are increasing in k for $\theta < \theta^*$ and decreasing in k for $\theta > \theta^*$.

It is worth highlighting several implications of Proposition 2. In an open election, the higher-valence candidate always wins, as $U_0(\theta)$ is increasing in θ. Under stationary term limits—which make seniority irrelevant—the higher-valence candidate also wins even in closed elections, so the system incentivizes the voter to retain talent efficiently, up to the constraint imposed by the term limits. However, under classic (or, more generally, non-stationary) term limits, the lower-valence candidate may win in a closed election. As illustrated in Figure 1b, when both candidates are weak ($\theta < \theta^*$), the voter favors the incumbent because she has a shorter maximum tenure, so she can do less harm. When both candidates are strong ($\theta > \theta^*$), the voter favors the challenger, who can be retained longer. This can be interpreted as a form of endogenous incumbency advantage (or disadvantage) created by term limits.

We turn now to an analysis of optimal term limits. To build intuition, consider Equations 4 and 5 in the two extreme cases $\mu = 0$ and $\mu = 1$. If $\mu = 0$ (politicians are always unbiased), in each election, the voter gets to choose the *higher* of the two candidates' electoral strengths. As a result, the voter's continuation utility increases over time, as she is able to find and retain better politicians over time who enter and compete. In this case term limits only serve to discard highly-selected incumbents and so are detrimental to welfare.

On the contrary, if $\mu = 1$ (politicians are always biased), in each election, the voter's utility is the *lower* of the two candidates' electoral strengths, since the winner fully extracts the difference between electoral strengths as policy rents. Term limits then become beneficial, because by limiting the candidates' maximum time in power, they reduce the implicit time horizon that the candidate and the voter are bargaining over in each election, and hence the leverage that the winner holds over the voter. To see this, consider, for example, a voter choosing between two candidates of ability $\theta_1 > \theta_2$ in a world of no term limits vs. a one-term limit. With a one-term limit, the cost of choosing the weaker candidate is at most $\theta_1 - \theta_2$. With no term limits, it may be as high as $\frac{\theta_1 - \theta_2}{1-\delta}$. In the case of p-stationary term limits, we can show directly that differences in electoral strengths are amplified as term limits are weakened (by part (iii) of Proposition 2, $U'(\theta)$ is increasing in p).

An alternative way of thinking about this case is that the voter's utility is *as if* politicians always offered $x_{it} = 0$, but the voter was forced to pick the weaker candidate every time. In such a world, the ability of the incumbent would *decline* over time until term limits are triggered; restarting the process as often as possible would then benefit the voter.

To summarize, optimal term limits solve a trade-off between optimal retention and curbing the bargaining power of strong candidates. The next Proposition provides a characterization of optimal term limits following from this intuition. We consider a term limit optimal if it maximizes the voter's ex ante utility V.

Proposition 3.

(i) If $\mu > \frac{1}{2}$, a one-term limit ($p_1 = 0$) is optimal among all stationary term limits.

(ii) If $\mu < \frac{1}{2}$, no term limits ($p_k \equiv 1$) is optimal among all stationary term limits.

(iii) If $\mu = \frac{1}{2}$, all term limits $(p_k)_{k\in\mathbb{N}}$ that are either stationary or finite horizon are welfare-equivalent.

Parts (i) and (ii) show that our intuition leads to a knife-edge result: when politicians are more often biased than not, even by a little, a one-term limit is optimal; conversely, when they are more often unbiased than not, having no term limits is optimal; when they are biased exactly half the time, all term limits are equally good. Finally, note that while parts (i) and (ii) are proven only for stationary limits, part (iii) also applies to *all* term limits with finite horizon.

4 Strategic Challenger Entry

In this section, we extend the baseline model to allow for strategic challenger entry. Specifically, we will replace the exogenous generation of challengers (part 2 of the stage game as described in Subsection 2.2) with the following assumption. Each party needing a challenger in a period t now generates a *potential* challenger who can choose to run or not.[14] Her ability is drawn from the distribution F but initially unknown, to herself and others. If she runs, she becomes her party's nominee, and her ability is publicly revealed. She pays a cost $c > 0$ of running and her payoffs net of this cost are given by Equation 2. If she does not run, her payoff is zero and her party runs a fill-in candidate with ability equal to zero. After the challengers and their abilities have been revealed, the stage game continues with part 3 as in Subsection 2.2. For convenience we will assume that if a potential challenger does

[14]In an open election, both potential challengers make their entry decisions simultaneously.

not enter, she misses her chance and another potential challenger will take her place the next time her party needs a challenger.[15]

The inclusion of strategic entry in the model adds two considerations to the search for optimal term limits which interact with the intuition set out in Section 3. On the one hand, strong incumbents discourage challenger entry. Their ability to extract rents is thus exacerbated: not only can stronger incumbents extract more rents against a challenger of fixed ability, but they also face worse challengers. Term limits that can remove such incumbents, then, become more desirable.[16] On the other hand, term limits themselves also *discourage* entry by limiting the candidates' maximum potential rents from running.

We will need to introduce some notation at this point. $q_k(\theta)$ will be a (potential) challenger's probability of running in a closed election against an incumbent of ability θ and seniority k. q_0 will be a (potential) challenger's probability of running in an open election. $r_k(\theta, \theta')$ will be the probability that a challenger of ability θ' wins against an incumbent of ability θ and seniority k.[17]

We will begin with the observation that more attractive incumbents face less competition. Formally, take a symmetric Markov strategy profile as given. Suppose there is an incumbent with ability θ and seniority k. If a challenger i runs with probability q, her payoff is $qT_k(\theta) - cq$, where $T_k(\theta)$ is i's expected rents and policy payoffs from running. Clearly i chooses $q_k(\theta) = 1$ if $T_k(\theta) > c$ and $q_k(\theta) = 0$ if $T_k(\theta) < c$; she is indifferent if $T_k(\theta) = c$. We can show the following:

Lemma 1. *If $U_k(\theta) < U_{\tilde{k}}(\tilde{\theta})$, then either $T_k(\theta) > T_{\tilde{k}}(\tilde{\theta})$ or $T_k(\theta) = T_{\tilde{k}}(\tilde{\theta}) = 0$.*

The incumbent's strength has two effects on the challenger: it makes the challenger less likely to win her first race, and conditional on the challenger being strong enough to win, it constrains what policies she can offer in her first term. Both effects reduce the expected gains from running.

[15] If we instead assume that the same agent will be the potential challenger the next time one is needed (until she eventually runs), challengers will be less willing to run if they expect a more favorable electoral landscape in the future. The intuition is otherwise unchanged.

[16] That removing a strong incumbent may benefit voters by encouraging entry is reminiscent of Baye, Kovenock and De Vries (1993), where a politician can extract higher bids from lobbyists by excluding those known to have the highest willingness to pay.

[17] These definitions are needed because the equilibria in this case may involve mixed strategies.

4.1 Equilibrium Characterization for Stationary Term Limits

We will now move towards an equilibrium characterization of the game with strategic entry. We begin by focusing on the special case of p-stationary term limits, for which we can fully characterize the set of SSMPE, i.e., the set of SMPE with U_k, V_k, q_k, r_k independent of k.

First we settle a basic question: is electoral strength $U(\theta)$ increasing in θ, as in Section 3? The answer is not obvious because, while higher-θ candidates directly produce higher voter utility, they can also extract more rents—especially if they drive out competition. In principle, if the entry deterrence effect is strong enough, it could make high-θ candidates less attractive *ex ante*, i.e., it could make $U(\theta)$ nonmonotonic. It turns out that this reversal is impossible under stationary term limits:

Proposition 4. *In any SSMPE with p-stationary term limits, $U(\theta)$ is weakly increasing.*

The logic of this result is simple. If we had $U(\theta) < U(\theta')$ for some $\theta > \theta'$, the candidate with ability θ would be *easier* to beat in subsequent elections, which would lead to *more* competition against her, not less; this, in turn, would make her more desirable. Thus, high θ cannot drive out so much competition that it becomes an electoral liability.[18]

However, note that U does not have to be *strictly* increasing—in fact, in equilibrium, U will typically be flat over some interval. As a result it will be important to assume something about how voters break ties if both candidates are equally attractive. We will assume that, in this case, voters flip a coin, i.e., $r(\theta, \theta') = \frac{1}{2}$, with one exception:[19] if one of the candidates is a fill-in challenger, the voter breaks ties in favor of the other candidate, i.e., $r(0, \theta) = 1$ and $r(\theta, 0) = 0$ even if $U(0) = U(\theta)$.[20]

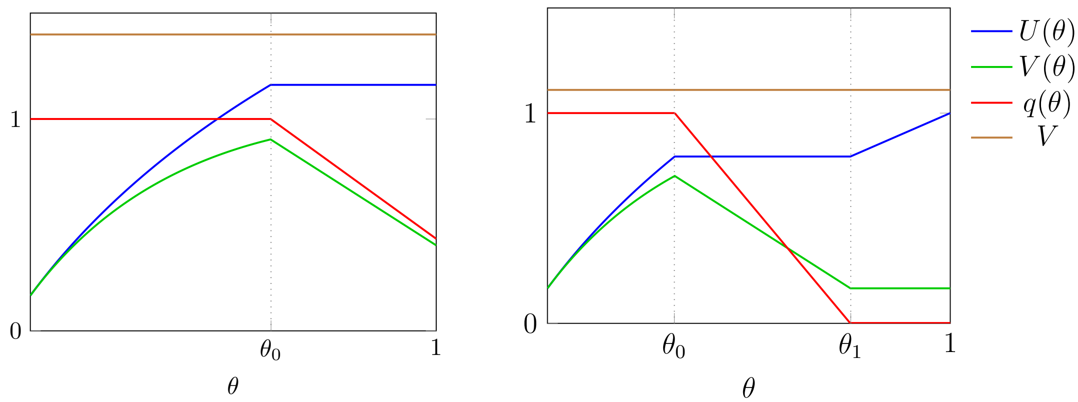
The next Proposition provides a full characterization of the SSMPE.

Proposition 5. *Any SSMPE of the game with strategic entry and p-stationary term limits is given by thresholds $0 \leq \theta_0 \leq \theta_1 \leq 1$ such that challengers enter against*

[18] We will see that this result does not carry through with non-stationary limits.

[19] This would be the uniquely selected outcome if a small, random symmetric payoff shock differentiating the two candidates was realized before the election but after platforms were chosen. Without assuming such a perturbation, other tie-breaking rules are possible and may induce more or less entry deterrence.

[20] This tie-breaking rule prevents the equilibrium from changing discontinuously when U becomes flat near zero (in the language of Proposition 5, when we switch between type 2 and type 3 equilibria). It would be uniquely selected if fill-in challengers' abilities were in fact slightly below zero.



(a) Type 1 equilibrium

(b) Type 2 equilibrium

Figure 2: Equilibrium under stationary term limits

incumbents of ability $\theta < \theta_0$, are indifferent against $\theta \in (\theta_0, \theta_1)$, and do not enter against $\theta > \theta_0$.

An SSMPE can be of type 1, 2, 3 or 4.

(i) In a type 1 equilibrium, $0 < \theta_0 < \theta_1 = 1$.

(ii) In a type 2 equilibrium, $0 < \theta_0 < \theta_1 < 1$.

(iii) In a type 3 equilibrium, $0 = \theta_0 < \theta_1 < 1$.

(iv) In a type 4 equilibrium, $0 = \theta_0 = \theta_1$.

In any SSMPE of any type:

(i) Within $[0, \theta_0)$, $U(\theta)$ and $V(\theta)$ are strictly increasing and $q(\theta) = 1$.

(ii) Within (θ_0, θ_1), $U(\theta)$ is constant, and $V(\theta)$ and q are linearly decreasing.

(iii) Within $(\theta_1, 1]$, $q(\theta) = 0$, and $U(\theta)$ and $V(\theta)$ are linearly increasing.[21]

Given values of all the parameters except c, we say a strategy profile is a candidate equilibrium *if it is an SSMPE for some value of c. Then, for each $\theta \in (0, 1)$, there is a unique candidate equilibrium with $\theta_0 = \theta$, and a unique candidate equilibrium with $\theta_1 = \theta$. In all cases $U(\theta)$, $V(\theta)$, V and $q(\theta)$ can be explicitly calculated.*

Moreover, given values of δ, p, μ, I, b, c and λ, there is $\phi > 1$ such that, if $\frac{f(\theta)}{1-F(\theta)} \leq \frac{\phi}{1-\theta} \forall \theta$, then there is $\gamma_0 > 0$ such that for all $\gamma < \gamma_0$, then the SSMPE of the game with these parameters is unique.

The intuition is as follows. When the incumbent is weak ($\theta < \theta_0$), she always faces competition. Hence, $U(\theta)$ and $V(\theta)$ are increasing in θ in this region, as a

[21]Unless $\mu = 1$, in which case $V(\theta)$ is constant.

stronger incumbent is unambiguously better for the voter. (In fact, the expressions for $U'(\theta)$ and $V'(\theta)$ in this region are the same as in Proposition 2.) However, when the incumbent's ability crosses above θ_0, challengers are sufficiently deterred that they are indifferent about running, as the potential rents from winning are harder to attain. If an incumbent of ability θ is not challenged, $V(\theta)$ is correspondingly lower, and hence so is $U(\theta)$. In equilibrium, the challenger's probability of running, $q(\theta)$, declines exactly at the rate needed to make $U(\theta)$ constant in this region, and challengers are indifferent about competing against any $\theta \in [\theta_0, \theta_1]$, which enables mixing. Since $U(\theta) = \theta + \delta p V(\theta) + \delta(1-p)V$ is constant, it follows that $q(\theta)$ must also be decreasing exactly at the rate needed to make $V(\theta)$ decrease at a rate $\frac{-1}{\delta p}$. (Having $U(\theta)$ be constant over an interval is necessary: if $q(\theta)$ declined any faster, $U(\theta)$ would be decreasing, contradicting Proposition 4; or, if $q(\theta)$ declined more slowly, $U(\theta)$ would be increasing and hence $T(\theta)$ would cross over c at a single point, so that q should jump discontinuously from 1 to 0 at θ_0, a contradiction.)

Figures 2a and 2b illustrate equilibria of types 1 and 2 respectively. The equilibrium is type 1 if entry is cheap enough that even the strongest incumbents are challenged with positive probability. It is type 2 if the cost of entry is intermediate, so weak incumbents are always challenged, while strong ones are never challenged—they can only be removed by term limits. The equilibrium is type 3 if entry is so costly that even the weakest incumbents are not always challenged.

That $U(\theta)$ is constant, but $V(\theta)$ decreasing, for $\theta \in (\theta_0, \theta_1)$ reflects the *intertemporal* differences between payoffs offered by candidates of different abilities. In this region, higher-θ candidates offer higher flow payoffs today, which exactly offset their higher likelihood of exploiting voters in the future. In other words, they offer more front-loaded payoff paths.[22]

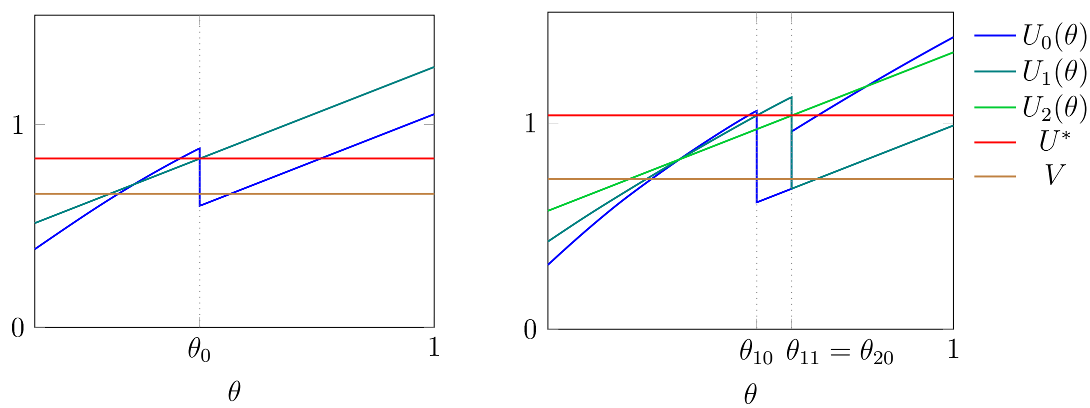
4.2 Equilibrium Characterization for General Term Limits

As in the baseline model, once we move away from stationary term limits, electoral strength depends on both seniority and ability. Thus, there may be incumbency advantage (or disadvantage) in the sense that when two candidates with the same θ compete, the incumbent may be preferred simply by virtue of being the incumbent or

[22]This is reminiscent of entrenched presidents in weak democracies: strong incumbents perform well at first, but over time become more corrupt. This pattern appears in the model despite the incumbent having no power to manipulate institutions.

vice versa.[23] When entry is strategic, a second form of non-monotonicity arises: $U_0(\theta)$ may be non-monotonic, i.e., even between two challengers, the voter may purposefully choose the one with lower ability due to fears that the stronger one will exploit her in the future.

The next Proposition gives a partial equilibrium characterization in this setting.



(a) Two-term limit

(b) Three-term limit

Figure 3: Equilibrium under classic term limits

Proposition 6. *In the game with strategic entry, assume* $p_m = 0$ *for some* m, $p_{m'} > 0$ *for all* $m' < m$. *Then all SMPEs are as follows. There is a utility threshold* U^* *such that* $q_k(\theta) = 1$ *if* $U_k(\theta) < U^*$ *and* $q_k(\theta) = 0$ *if* $U_k(\theta) > U^*$.

For each $k = 0, \ldots, m-2$ *there is a set of cutoffs* $A_k = \{\theta_{1k}, \ldots, \theta_{l_k k}\}$ *such that* $A_{k-1} \supseteq A_k$ $\forall k$, *and* $l_k \leq 2^{m-k-1} - 1$ *for* $k \leq m-2$. *Cutoffs* $\theta \in A_k$ *are given by the condition* $U_{k'}(\theta) = U^*$ *for some* $k' > k$. *Electoral strength* $U_k(\theta)$ *has discrete jumps (up or down) at the cutoffs* $\theta_{ik} \in A_k$ *and is smoothly increasing in between.*

By way of an illustration, we show in more detail the special case of a classic two-term limit:

Corollary 1. *Consider the game with strategic entry and a 2-term limit. Then all SMPEs are as follows. There is a utility threshold* U^* *such that* $q_1(\theta) = 1$ *if* $U^* >$

[23] Our model contains two other features that might also be construed as *incumbency advantage*. First, incumbents are a selected group of politicians who have won at least one election, so they tend to have higher ability. Second, with strategic entry, strong incumbents tend to face weak challengers. Both forces increase the odds of reelection. But they do not give an advantage, conditional on facing a challenger of equal ability. What we call incumbency advantage gives incumbents an edge against a challenger of equal ability, and is a unique consequence of non-stationary term limits.

$U_1(\theta)$ and $q_1(\theta) = 0$ if $U^* < U_1(\theta)$. In the nondegenerate case where $U_1(0) < U^* < U_1(1)$, the SMPE is described by a cutoff $\theta_0 \in (0, 1)$ and a probability q_0 of entering open elections, such that:

(i) $U^* = U_1(\theta_0)$, *so* $q_1(\theta) = 1$ *for* $\theta < \theta_0$ *and* $q_1(\theta) = 0$ *for* $\theta > \theta_0$.

(ii) $U_1(\theta) = \theta + \delta V$ *is linearly increasing.* $U_0(\theta)$ *is smoothly increasing everywhere except at* θ_0, *where it has a discrete drop.*

(iii) $U_1'(\theta) < U_0'(\theta)$ *for* $\theta < \theta_0$ *and* $U_1'(\theta) \leq U_0'(\theta)$ *for* $\theta > \theta_0$ *(with equality only if* $\mu = 1$*).*

(iv) If μ *is close enough to 1, then* $U_1(\theta) \geq U_0(\theta)$ *for* $\theta > \theta_0$, *and* $U_1(0) \geq U_0(0)$, *i.e., there is incumbency advantage for high ability as well as for very low ability candidates. There may be incumbency advantage or disadvantage at intermediate ability.*

Two degenerate cases are possible. For c low enough, $U^* > U_1(1)$ and challengers always enter. For c high enough, $U^* < U_1(0)$ and challengers never enter an open election.

Figures 3a and 3b show SMPEs in examples with a two-term limit and a three-term limit, respectively. The logic behind the non-monotonicity of $U_k(\theta)$ is most easily explained in the special case of a two-term limit. Just as in the stationary case, incentives to run against an incumbent (θ, k) are a decreasing function of $U_k(\theta)$, so there is a threshold U^* such that challengers run against incumbents with $U < U^*$ and not against ones with $U > U^*$. However, now $U_1(\theta)$ cannot be constant over an interval, since the continuation utility from electing a term-limited incumbent is fixed: $U_1(\theta) \equiv \theta + \delta V$. In particular, $U_1(\theta)$ must cross U^* at a single point. Then incumbents with $\theta < \theta_0$ are always challenged while those with $\theta > \theta_0$ never are. This results in $U_0(\theta)$ having a discontinuous drop at θ_0: challengers with θ just above θ_0 are expected to suppress so much competition in their second term that they are *ex ante* less desirable than an alternative with θ just below θ_0. For higher m, the equilibrium becomes more complicated as utilities "cycle" around U^*, by the same logic: if $U_k(\theta) > U^*$, then $q_k(\theta) = 0$ which often implies $U_{k-1}(\theta) < U^*$, $q_{k-1}(\theta) = 1$ which in turn often implies $U_{k-2}(\theta) > U^*$ and so on.

Another consequence of this analysis is that, with strategic entry, incumbency advantage may be non-monotonic. In particular, under a two-term limit, seniority is favored when both candidates are strong ($\theta, \theta' > \theta_0$) or when they are very weak (θ, θ' near zero), though not necessarily in the middle (part (iii) of Corollary 1). This contrasts with the logic from Section 3, in which seniority was only valuable for weak candidates. The broad conclusion is that the potential for non-stationary term limits to incentivize suboptimal talent retention by the voter is amplified in a world of strategic entry.

4.3 Optimal Term Limits

Finally, we provide a short discussion of how the optimal term limits change in a world of strategic entry as we vary two parameters of the model: the cost of entry, c, and the probability of politicians being biased, μ. Note that as $c \to 0$, the model converges to the case of exogenous challenger entry (Section 3).

To summarize the insights from Sections 3 and 4, term limits affect the voter's welfare, V, through four channels. First, term limits prevent optimal retention of talented incumbents. Second, they limit the scope for rent extraction by limiting the implicit bargaining horizon of each election. (These two effects are present even when entry is exogenous.) Third, they increase entry by removing from office incumbents who would otherwise deter competition and be reelected indefinitely. Fourth, they decrease entry by limiting a challenger's maximum tenure, hence her expected rents. (These two effects are specific to the case of strategic entry.)

Term limits increase welfare through the second and third effects and decrease it through the first and fourth. It is difficult to determine in general which effects dominate. We give some partial results and intuition supported by simulations.

The following Proposition is a partial analog of Proposition 3 for the case of strategic entry.

Proposition 7. *Let $c_0 = \frac{b}{2} > 0$. Then, if a one-term limit is welfare-superior to another term limit $(p_k)_{k\geq 1}$ in the game with exogenous entry, a one-term limit is also welfare superior to $(p_k)_{k\geq 1}$ under strategic entry whenever $c < c_0$. In particular, if $\mu \geq \frac{1}{2}$, a one-term limit is optimal among all stationary term limits if $c < c_0$.*

Proposition 7 says that, compared to the exogenous entry case, the scales tilt further in favor of a one-term limit when c is positive but small enough. The reason is that, so long as $c < \frac{b}{2}$, it is always worthwhile to run in open elections, that is, $q_0 = 1$; a one-term limit eliminates the possibility of entry deterrence (incumbents never run for reelection so they can't deter entrants) and gives the same welfare as when $c = 0$, whereas other term limits must become weakly worse (compared to when $c = 0$) due to deterrence.

Different forces are at work for high c. When c is high enough that challengers drop out of open elections, weak term limits may be needed to incentivize entry by increasing the potential rewards from running. In fact, for high c, the optimal term limits may be weaker than under exogenous entry, because the need to boost rewards eventually dominates concerns about limiting rent extraction.

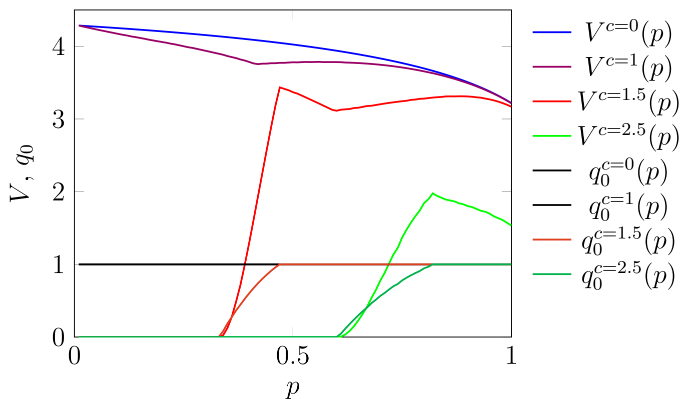
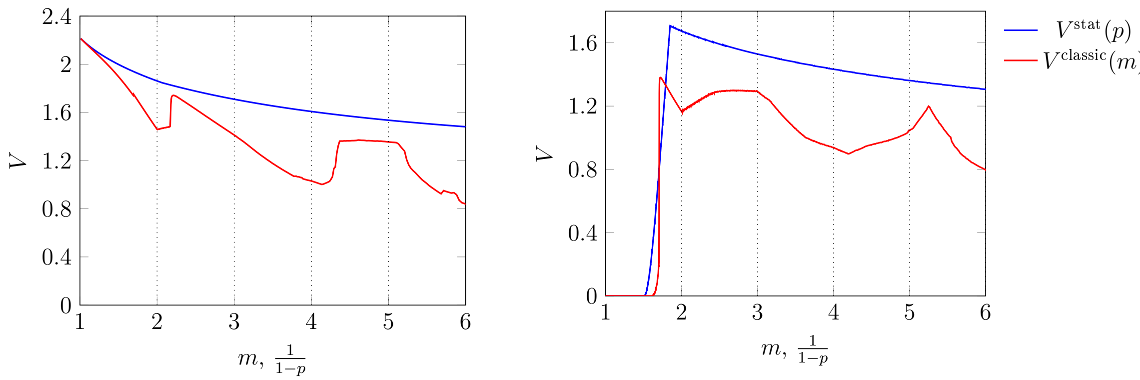


Figure 4: Welfare and entry as a function of p for $b = 2$, $\mu = 1$, $\delta = 0.85$, $c \in \{0, 1, 1.5, 2.5\}$, $f(\theta) = 3\theta^2$

Figure 4 shows numerical results which illustrate this intuition, showing how V varies under p-stationary limits as a function of p for different values of c, under the assumption $\mu = 1$. First, for μ close to 1, a one-term limit ($p = 0$) is optimal even without strategic entry ($c = 0$), but becomes even more desirable when c is intermediate (compare $V^{c=0}$ to $V^{c=1}$). Second, regardless of μ, a high p (all the way up to $p = 1$) may be optimal when c is very high, as challengers must be enticed to enter open elections.

Finally, in Figure 5, we illustrate numerical results comparing the welfare properties of stationary and classic term limits as c varies.[24] (Stationary limits are plotted by mapping p to $\frac{1}{1-p}$, the expected maximal tenure. To fill in the graph for classic term limits, we use "fractional" $(m + p)$-limits where m terms are allowed, followed by an $(m + 1)$-th term with some probability p.) The results of simulations show

[24]For stationary term limits, the numerical solution can be computed using the analytical results in the paper. For classic term limits, the simulation constructs a solution as in Proposition 6 and iterates on U^*, V and U_k until convergence is reached. For all parameters used the limit is unique.



(a) $c = 0.5$

(b) $c = 1.5$

Figure 5: Welfare under stationary and classic term limits; $b = 2$, $\mu = 1$, $\delta = 0.85$, $c \in \{0.5, 1.5\}$, $F \sim U[0, 1]$

that, except for when a one-term limit is optimal (attainable as either $p = 0$ or $m = 1$), stationary term limits are welfare-superior to classic term limits. A tentative intuition is that this is because classic term limits create stronger incentives for sub-optimal retention (because the functions $U_k(\theta)$ depend on k and are non-monotonic), and because they generate incumbency advantage that deters entry against strong incumbents even more than in the stationary case.

One last observation concerning welfare effects is that, even when the optimal term limits with and without strategic entry coincide, the intertemporal considerations often differ. Under exogenous entry, $V_k(\theta)$ is always increasing in θ—it is always preferable to go into a closed election with a stronger incumbent. Thus, in the absence of term limits, the voter's continuation payoff increases over time, converging to $V(1)$ as $t \to \infty$. Hence, a social planner more patient than the voter ($\overline{\delta} > \delta$) may favor weaker term limits than the voter wants. When entry is strategic, in contrast, $V_k(\theta)$ is non-monotonic. In the extreme case of $\mu = 1$ and no term limits, if the equilibrium is type 2 or 3, then the voter's continuation payoff eventually converges to $V(1) = 0$. Hence, a patient social planner would favor stronger term limits than the voter.

5 Flexible Term Limits and Regime Change

So far, we have focused on the *ex ante* choice of fixed term limits. This represents voters (or a social planner) choosing a constitution before the game begins, and before the likely incumbents are known; afterwards, the constitution is unchangeable.

However, many attempts to change term limits occur in a different context. The push for reform is often driven by a popular incumbent that would need a constitutional amendment to run again. For example, Hugo Chávez came to power in Venezuela in 1999. In 2009, during his purported last term, he abolished term limits in a referendum, after which he was reelected in 2013. In Argentina, prior to 1994, presidents were restricted to a single six-year term. Elected in 1989, Carlos Menem engineered a constitutional reform that changed the maximum to two four-year terms, allowing him to run again in 1995. During his second term he campaigned unsuccessfully to extend the limit to three terms. Similarly, Fernando Cardoso in Brazil and Álvaro Uribe in Colombia extended their respective limits from one four-year term to two.[25] The common theme in these examples is that voters know who the incumbent will be if the term limits are relaxed.

In this section, we sketch a version of the model allowing for this sort of discretional change of rules. For brevity we will restrict ourselves to the case $\mu = 1$. Concretely, we will model *flexible term limits* as follows: the default rule is a one-term limit but, after *each* election, the voter can choose in a referendum to override the rule and let the incumbent run for reelection *this time*; the incumbent's right to run again must be renewed every time.[26]

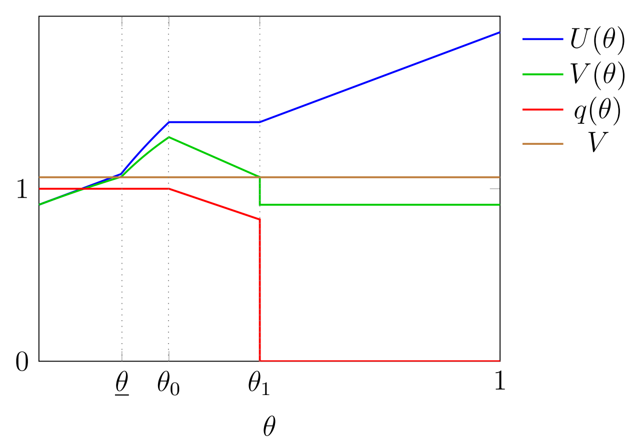


Figure 6: Equilibrium with flexible term limits

[25]For an in-depth discussion of the Latin American case, see Carey (2003).

[26]A possible alternative is *persistent regime change*: there is initially a one-term limit, but the incumbent in each period can call for a referendum to abolish term limits *permanently*. The general logic of the solution is the same, but this variant has different intertemporal implications, as incumbents may become entrenched once term limits are removed.

The way the voter uses flexible term limits is straightforward. Since she cannot commit, she lets an incumbent of ability θ run again iff $V(\theta) \geq V$. This leads to the following equilibrium:

Proposition 8. *Consider the game with flexible term limits and* $\mu = 1$. *The MPE may be of type 1, 2, or 3. In a type 1 equilibrium, there are* $0 < \underline{\theta} < \theta_0 < 1$ *such that* $0 < q(\theta) < 1$ *for* $\theta > \theta_0$ *and candidates can run again if* $\theta \in [\underline{\theta}, 1]$. *In a type 2 equilibrium, there are* $0 < \underline{\theta} < \theta_0 < \theta_1 < 1$ *such that* $q(\theta) = 1$ *for* $0 \leq \theta \leq \theta_0$, $q(\theta)$ *is decreasing between* θ_0 *and* θ_1, *and drops discontinuously to* $q(\theta) = 0$ *for* $\theta > \theta_1$. *Candidates are allowed to run again if* $\theta \in [\underline{\theta}, \theta_1]$. *In a type 3 equilibrium,* $q \equiv 0$.

In general the voter discards bad politicians (with $\theta < \underline{\theta}$) and keeps good ones. However, when competition against strong candidates is low (in a type 2 equilibrium), the best candidates (with $\theta > \theta_1$) are also discarded as they are, in effect, dangerous. If allowed to run, such candidates would deter entrants and thus leave the voter no choice but to reelect them; hence, the voter will bar their reelection bid in the first place.[27] This is illustrated in Figure 6.

In terms of welfare, flexible term limits may be better or worse than the fixed term limits from the main model, because the two types of schemes differ in two ways. On the one hand, flexible term limits allow the voter to condition on θ—they grant the freedom to target undesirable incumbents only. On the other hand, the voter, being unable to commit, decides who to keep based on the "greedy" rule $V(\theta) > V$, ignoring that such decisions affect challengers' incentives to enter. Thus, when weak term limits would be *ex ante* optimal (because the prize must be increased to promote entry), the voter will still be tempted to kick out entrenched incumbents. But challengers will anticipate this and choose not to run. If anything, the sin of flexible term limits is to give voters *too much* freedom to avoid exploitation.

6 Conclusions

We study the optimal design of term limits in a world where politicians are tempted to choose policies disliked by the median voter, and where challengers may be less likely

[27] This is reminiscent of Athenian ostracism, whereby citizens could preemptively exile other citizens that might become too powerful and pose a threat to the state (Aristotle and Rackham, 1935, chap. 22).

to run if their probability of winning is low. Although the interplay between term limits and welfare outcomes is complex, the model highlights a few general themes.

First, term limits may be valuable as a tool for disciplining incumbents, even *without* strategic entry: just by bringing the continuation values from different candidates closer together, term limits can reduce opportunities for exploitation and improve welfare.

Second, the presence of strategic entry usually increases the need for term limits—in particular, when the cost of entry is intermediate, so that challengers are willing to enter open elections but unwilling to face a strong incumbent (a plausible case). However, the result is reversed when the cost is very high: in that case term limits should be relaxed so as to entice politicians to compete for a bigger prize.

Third, the structure of term limits matters. Conventional term limits induce voters to condition on seniority and generate artificial incumbency advantage or disadvantage. In particular, a two-term limit makes strong incumbents unbeatable in their reelection bid, even by equally qualified challengers, which in turn discourages entry and lowers welfare. For these reasons, stationary term limits are often welfare-superior. Naturally, stochastic term limits are difficult to implement in practice.

We conclude with some comments about "robustness checks" and avenues for further work.

The assumption of quadratic policy payoffs for the politicians is only for simplicity; we could use any concave function. As for the voter, we could relax either the assumption of quadratic utility or the assumption of a single voter, but not both. The reason is that, with a continuum of voters, the median voter is always decisive (hence, equivalent to a representative voter) in this dynamic setting if the voters have quadratic preferences (see Banks and Duggan, 2006), but not otherwise. The assumption that candidates only care about policies while they are in office simplifies some of the proofs of Proposition 5 and Lemma 1 but is not essential; if we assume candidates care about policies even after they lose or fail to run, all the results go through, with the caveat that Lemma 1 would only hold for γ close enough to zero. (For large γ, a challenger may want to run against a strong incumbent to prevent very bad policies from being implemented.)

We take seriously the notion that voters are forward-looking; this assumption is central to the results. It is plausible that voters would pay attention to expected continuations *sometimes* in practice, but perhaps the model expects too much rationality from them. Assuming that the voter is myopic (formally, $\delta_v = 0$) results in a model that is quite tractable but yields very different results. In that case, with exogenous entry, no term limits are always optimal, because the policy rents a winner can extract are independent of continuation values. In contrast, when entry is strategic, term limits become important, because the social planner is now more concerned about incumbents becoming entrenched, and voters do not protect against this. Finally, talent retention is always optimal (the higher-θ candidate always wins); in particular, non-stationary limits no longer generate incumbency advantage or disadvantage.

Our model intentionally does not assume that incumbents can build up ability or *de facto* power over time, in order to isolate effects driven solely by "fair" electoral competition. But this is a relevant force in practice and worth integrating into the framework.

Finally, while in this paper we consider term limits as a solution to political agency problems, other institutional innovations may be helpful as a complement or substitute to them. For example, the incumbent could be required to perform at a certain level to be able to stand for reelection (Gersbach and Liessem, 2008) or to earn a certain supermajority of the vote to be reelected (Gersbach and Müller, 2017). However, no such addition to the model is likely to bring us all the way to the first-best ($x_t = 0$ for all t and challengers always enter) unless the institution can somehow condition finely on the candidates' abilities.

A Appendix

Proof of Proposition 1. Note that if $U_{k_i}(\theta_i) - \lambda x_{it}^2 > U_{k_j}(\theta_j) - \lambda x_{jt}^2$, then the voter must elect i, and vice versa. This follows from the definition of $U_k(\theta)$.

Suppose that i is from party L and j from R. If i is biased, we can verify that the described actions are compatible with equilibrium: given that $x_{jt} = 0$ and the voter is assumed to choose i when indifferent, i can win by offering any x such that $U_{k_i}(\theta_i) - \lambda x^2 \geq U_{k_j}(\theta_j)$, or equivalently $|x| \leq \sqrt{\frac{U_{k_i}(\theta_i) - U_{k_j}(\theta_j)}{\lambda}}$. Then i's optimal offer is the minimum of this set, which leaves the voter indifferent: $x^* = -\sqrt{\frac{U_{k_i}(\theta_i) - U_{k_j}(\theta_j)}{\lambda}}$.[28] Given i's offer, j cannot win, so she is indifferent between all her actions.

It is also an equilibrium for j to choose any mixed strategy that makes x^* optimal for i, with i's and the voter's strategies unchanged. These equilibria are payoff-equivalent to the one presented. There are no equilibria in which i plays x^* and does not always win—if the voter elected j with positive probability on the equilibrium path, i would deviate closer to 0.

Suppose now that there is a (possibly mixed) equilibrium in which i's strategy is not to choose x^* w.p. 1. Because offering any $x \in (x^*, -x^*)$ guarantees a win with payoff $b - \gamma(x + I)^2 + \delta W$ (where W is a shorthand for i's payoffs from future periods), i's payoff from any policy in the support of her strategy must be at least $b - \gamma(x^* + I)^2 + \delta W$. Policies $x < -I$ are also dominated by $-I$. Hence the support of i's strategy must be contained in $[-I, x^*]$. Let $P > 0$ be the probability that i plays $x < x^*$, and let $\underline{x}$ be the infimum of the support of i's strategy. j can guarantee a payoff $(b - \gamma I^2)P > 0$ by offering $x_{jt} = 0$, hence any policy in the support of her strategy must pay at least this much. Let $\overline{x} \in [0, I]$ be the supremum of the support of j's strategy. If $U_{k_i}(\theta_i) - \lambda \underline{x}^2 > U_{k_j}(\theta_j) - \lambda \overline{x}^2$ then j would get payoff 0 from playing any policy in a neighborhood of $\overline{x}$, a contradiction; if the reverse strict inequality holds, i would get 0 from playing any policy in a neighborhood of $\underline{x}$, a contradiction. If there is equality, for i, j to both get payoffs bounded away from zero from playing strategies near $\underline{x}$, $\overline{x}$ respectively, it would have to be that they are both playing $\underline{x}$, $\overline{x}$ respectively with positive probability. But then both would profit by deviating to policies slightly closer to zero, a contradiction.

Finally, if i is unbiased, then she can both guarantee a win and get her highest

[28] This is also better than offering a more extreme x, since losing gives i a payoff of 0, while winning pays at least $b - \gamma I^2 > 0$.

possible flow payoff at time t by offering $x_{it} = 0$. j's strategy is irrelevant. $\square$

Proof of Proposition 2. (i) The existence and uniqueness follow from applying the Contraction Mapping Theorem to the argument presented in the text. Briefly, let $\hat{U}_0^1$, $\hat{U}_0^2$ be two conjectured values of U_0, drawn from the domain of bounded Lebesgue-measurable functions from $[0, 1]$ to $\mathbb{R}$, and denote $\|\hat{U}_0^1 - \hat{U}_0^2\|_\infty = \Delta$. Denote by U_k^1, U_k^2, V_k^1, V_k^2 the results of calculating U_k, V_k from Equations 3, 4 and 5 under the conjecture that $U_0 = \hat{U}_0^1$ or $\hat{U}_0^2$, respectively. We will use the following Lemma:

Lemma 2. *Let $a, a' : [0, 1] \to \mathbb{R}$ be Lebesgue-measurable bounded functions and H_1, H_2 be cdfs. Then, if $\|a - a'\|_\infty \leq \Delta$,*

$$|E\left[\max(a(x), a(y))|x \sim H_1, y \sim H_2\right] - E\left[\max(a'(x), a'(y))|x \sim H_1, y \sim H_2\right]| \leq \Delta, \quad \textit{and}$$
$$|E\left[\min(a(x), a(y))|x \sim H_1, y \sim H_2\right] - E\left[\min(a'(x), a'(y))|x \sim H_1, y \sim H_2\right]| \leq \Delta.$$

Additionally, for any $x_0 \in \mathbb{R}$,

$$|E\left[\max(x_0, a(y))|y \sim H_2\right] - E\left[\max(x_0, a'(y))|y \sim H_2\right]| \leq \Delta \quad \textit{and}$$
$$|E\left[\min(x_0, a(y))|y \sim H_2\right] - E\left[\min(x_0, a'(y))|y \sim H_2\right]| \leq \Delta.$$

Proof. Because $\|a - a'\|_\infty \leq \Delta$, $a(z) \leq a'(z) + \Delta$ for all z. Hence $\max(a(x), a(y)) \leq \max(a'(x) + \Delta, a'(y) + \Delta) = \max(a'(x), a'(y)) + \Delta$. Taking expectations,

$$E\left[\max(a(x), a(y))|x \sim H_1, y \sim H_2\right] \leq E\left[\max(a'(x), a'(y))|x \sim H_1, y \sim H_2\right]| + \Delta.$$

Applying the same argument with a and a' reversed yields the first inequality. The argument for minimum functions is analogous. The third and fourth inequalities are special cases with, e.g., $a(x) \equiv x_0$ and any H_1. $\square$

Suppose $p_m = 0$. Applying Lemma 2 to Equation 5, we get $|V^1 - V^2| \leq \Delta$; $\|U_{m-1}^1 - U_{m-1}^2\| \leq \delta\Delta$; $\|V_{m-1}^1 - V_{m-1}^2\| \leq \Delta$; $\|U_{m-2}^1 - U_{m-2}^2\| \leq \delta\Delta$; and eventually $\|U_0^1 - U_0^2\| \leq \delta\Delta$. Hence the mapping $\hat{U}_0 \mapsto U_0$ is a contraction.
(ii) This follows from showing that the mapping in part (i) preserves continuity and monotonicity (i.e., if $\hat{U}_0$ is continuous and (strictly) increasing, then U_0 is continuous and (strictly) increasing) and repeating the argument of part (i) with the domain restricted to the space of continuous and (strictly) increasing bounded measurable functions from $[0, 1]$ to $\mathbb{R}$.

(iii) Under p-stationary limits, Equations 3 and 4 become $U(\theta) = \theta + \delta p V(\theta) + \delta(1 - p)V$ and $V(\theta) = \mu E\left[\min(U(\theta), U(\theta'))|\theta' \sim F\right] + (1-\mu)E\left[\max(U(\theta), U(\theta'))|\theta' \sim F\right]$ respectively. Differentiating with respect to θ yields

$$\begin{aligned} U'(\theta) &= 1 + \delta p V'(\theta) \\ V'(\theta) &= U'(\theta)\left[\mu(1 - F(\theta)) + (1-\mu)F(\theta)\right] \end{aligned}$$

(Note that in calculating $V'(\theta)$ we use that $P(U(\theta) \geq U(\theta')|\theta' \sim F) = F(\theta)$, which follows from U being strictly increasing.) Rearranging we obtain $U'(\theta) = \frac{1}{1-\delta p[\mu+(1-2\mu)F(\theta)]}$ and $V'(\theta) = \frac{\mu+(1-2\mu)F(\theta)}{1-\delta p[\mu+(1-2\mu)F(\theta)]}$.

Next we calculate $U(0)$, $V(0)$ and V, which together with the above amounts to a full characterization of the value functions. By Equation 3, $U(0) = \delta p V(0) + \delta(1-p)V$. By Equation 4, $V(0) = \mu U(0) + (1-\mu)E(U(\theta')|\theta' \sim F)$. Plugging in Equation 3, and using that $E(V(\theta')|\theta' \sim F) = V$ by construction, this becomes

$$\begin{aligned} V(0) &= \mu(\delta p V(0) + \delta(1-p)V) + (1-\mu)E(\theta' + \delta p V(\theta') + \delta(1-p)V|\theta' \sim F) \\ V(0) &= \mu(\delta p V(0) + \delta(1-p)V) + (1-\mu)(E(\theta') + \delta V) \\ V(0) &= \frac{(\delta - \delta\mu p)V + (1-\mu)E(\theta')}{1 - \delta p\mu}, \quad U(0) = \frac{(\delta^2 p(1-\mu) + \delta(1-p))V + \delta p(1-\mu)E(\theta')}{1-\delta p\mu} \end{aligned}$$

We can now calculate V:

$$\begin{aligned} V &= E(V(\theta)|\theta \sim F) = \int_0^1 V(\theta)f(\theta)d\theta = V(1)F(1) - V(0)F(0) - \int_0^1 V'(\theta)F(\theta)d\theta \\ &= \int_0^1 V'(\theta)d\theta - \int_0^1 V'(\theta)F(\theta)d\theta + V(0) = \int_0^1 V'(\theta)(1-F(\theta))d\theta + \frac{(\delta - \delta\mu p)V + (1-\mu)E(\theta)}{1-\delta p\mu} \end{aligned}$$

$$\begin{aligned} V &= \frac{1-\delta p\mu}{1-\delta}\left[\int_0^1 V'(\theta)(1-F(\theta))d\theta + \frac{(1-\mu)E(\theta)}{1-\delta p\mu}\right] \\ V &= \frac{1-\delta p\mu}{1-\delta}\int_0^1 \frac{\mu + (1-2\mu)F(\theta)}{1-\delta p\left[\mu + (1-2\mu)F(\theta)\right]}(1-F(\theta))d\theta + \frac{(1-\mu)}{1-\delta}\int_0^1 (1-F(\theta))d\theta \\ V &= \frac{1}{1-\delta}\int_0^1 \frac{1-\delta p\mu + (1-\delta p)(1-2\mu)F(\theta)}{1-\delta p\left[\mu + (1-2\mu)F(\theta)\right]}(1-F(\theta))d\theta \end{aligned} \tag{6}$$

(iv) Suppose that $V_{m-1}(\theta) > V$. Then, by Equation 3, $U_{m-2}(\theta) > U_{m-1}(\theta)$. By Equation 4, $V_{m-2}(\theta) > V_{m-1}(\theta) > V$. Iterating, we find $V_k(\theta)$ is decreasing in k for all k. Analogously, if $V_{m-1}(\theta) < V$, then $V_k(\theta)$ is increasing in k.

Now note that if there is $\theta^* \in (0,1)$ with $V_{m-1}(\theta^*) = V$, then by the same argument $V_k(\theta^*) = V$ for all k and $U_k(\theta^*)$ is constant in k. And from part (ii) we know $V_{m-1}(\theta)$ is increasing in θ, so $V_{m-1}(\theta) > V_{m-1}(\theta^*) = V$ for all $\theta > \theta^*$ and $V_{m-1}(\theta) < V_{m-1}(\theta^*) = V$ for all $\theta < \theta^*$. The result follows.

To finish we must show that such a θ^* exists. Suppose not. Because V_{m-1} is continuous (part (ii)), either $V_{m-1}(\theta) \geq V$ for all θ or $V_{m-1}(\theta) \leq V$ for all θ. WLOG, suppose the former. Let $V_0(\theta)$ be the voter's expected utility from a hypothetical election between two challengers, one known to have ability θ and the other of unknown ability $\theta' \sim F$. The previous argument implies $V_0(\theta) \geq V$ for all θ, and by part (ii), $V_0(\theta) > V$ for all $\theta > 0$. But $V = E(V_0(\theta)|\theta \sim F)$, a contradiction. $\square$

Proof of Proposition 3. For parts (i) and (ii), note that the integrand in Equation 6 can be rewritten as

$$(1 - F(\theta)) \left[1 + \frac{(1-2\mu)F(\theta)}{1 - \delta p \left[\mu + (1-2\mu)F(\theta)\right]}\right].$$

Here $\mu + (1-2\mu)F(\theta)$ ranges between $F(\theta)$ and $1 - F(\theta)$ as μ ranges between 0 and 1, i.e., it is always positive, whereas $(1-2\mu)F(\theta)$ is positive when $\mu < \frac{1}{2}$ but negative when $\mu > \frac{1}{2}$. Then, for all θ, the derivative of this expression with respect to p is positive if $\mu < \frac{1}{2}$, zero if $\mu = \frac{1}{2}$ and positive if $\mu > \frac{1}{2}$. Hence, from Equation 6, the same is true of V.

This argument also proves part (iii) within the domain of stationary limits. For term limits with finite horizon, we leverage the fact that, if $\mu = \frac{1}{2}$, then $\mu \min(x,y) + (1-\mu)\max(x,y) \equiv \frac{x+y}{2}$. Then $V = E(U_0(\theta)|\theta \sim F)$ and for all θ, k, $V_k(\theta) = \frac{U_k(\theta)}{2} + \frac{E(U_0(\theta')|\theta' \sim F)}{2} = \frac{U_k(\theta)+V}{2}$. We now do backwards induction on k. Denote $\overline{\theta} = E(\theta|\theta \sim F)$ and $\overline{U}_k = E(U_k(\theta)|\theta \sim F)$. For $k = m-1$, $U_{m-1}(\theta) = \theta + \delta V$, so $\overline{U}_{m-1} = \overline{\theta} + \delta V$. Now suppose $\overline{U}_k = a_k \frac{\overline{\theta}}{1-\delta} + (1-a_k)V$ for some $a_k > 0$ (in particular $a_{m-1} = 1-\delta$). Then $\overline{U}_{k-1} = \overline{\theta} + \frac{\delta p_k}{2}\overline{U}_k + \delta\left(1 - \frac{p_k}{2}\right)V$, i.e., $\overline{U}_{k-1}$ takes the same form with $a_{k-1} = 1 - \delta + \frac{\delta p_k}{2} a_k$. Iterating, we end up with

$$V = E(U_0(\theta)|\theta \sim F) = \overline{U}_0 = a_0 \frac{\overline{\theta}}{1-\delta} + (1-a_0)V$$

for some $a_0 > 0$, whence $V = \frac{\overline{\theta}}{1-\delta}$. We can also check that this is the same welfare obtained under any p-stationary limit from Equation 6. $\square$

Proof of Proposition 4. We will first show that $U(\theta)$ is minimized at $\theta = 0$. For the sake of contradiction, suppose not. Let $\theta^* > 0$ be such that $U(\theta^*)$ is minimal.[29] Let $\Delta = U(0) - U(\theta^*)$. By Lemma 1, either $T(0) < T(\theta^*)$, in which case $q(0) \leq q(\theta^*)$, or $T(0) = T(\theta^*) = 0$, in which case $q(0) = q(\theta^*) = 0$—in particular $q(0) \leq q(\theta^*)$. Note that $U(\theta^*) = \theta^* + \delta p V(\theta^*) + \delta(1-p)V$, where

$$V(\theta^*) = \mu U(\theta^*) + (1-\mu)\left[E(U(\theta')|\theta' \sim F)q(\theta^*) + U(0)(1-q(\theta^*))\right]$$
$$\Longrightarrow U(\theta^*) = \frac{\theta^* + \delta p(1-\mu)\left[E(U(\theta')|\theta' \sim F)q(\theta^*) + U(0)(1-q(\theta^*))\right] + \delta(1-p)V}{1-\delta p\mu}.$$

(We have used that $\min(U(\theta^*), U(\theta')) \equiv U(\theta^*)$ and $\max(U(\theta^*), U(\theta')) \equiv U(\theta')$, by assumption.) Similarly $U(0) = \delta p V(0) + \delta(1-p)V$, where

$$V(0) \leq \mu U(0) + (1-\mu)\left[E(\max(U(0), U(\theta'))|\theta' \sim F)q(0) + U(0)(1-q(0))\right]$$
$$\Longrightarrow U(0) \leq \frac{\delta p(1-\mu)\left[E(\max(U(0), U(\theta'))|\theta' \sim F)q(0) + U(0)(1-q(0))\right] + \delta(1-p)V}{1-\delta p\mu}.$$

(We have used that $\min(U(0), U(\theta')) \leq U(0)$ and $\max(U(0), U(0)) = U(0)$.) Now note that $E(\max(U(0), U(\theta'))|\theta' \sim F) - E(U(\theta')|\theta' \sim F) \leq \Delta$ and $U(0) - E(U(\theta')|\theta' \sim F) \leq \Delta$. Then $U(0) - U(\theta^*) \leq \frac{-\theta^* + \delta p(1-\mu)q(\theta^*)\Delta}{1-\delta p\mu} < \Delta$, a contradiction.

Now suppose that there are $\theta < \theta'$ such that $U(\theta) > U(\theta')$ and denote $U(\theta) - U(\theta') = \Delta > 0$. By the same argument as before, $q(\theta) \leq q(\theta')$. Hence

$$U(\theta) - U(\theta') = \theta - \theta' + \delta p(V(\theta) - V(\theta')) < \delta p(V(\theta) - V(\theta'))$$

Now, if $q(\theta) = q(\theta')$, we obtain from Lemma 2 that $V(\theta) - V(\theta') \leq \Delta$, hence $U(\theta) - U(\theta') < \delta p\Delta$, a contradiction. If $q(\theta) < q(\theta')$, the same conclusion still holds because $V(\theta)$ is increasing in $q(\theta)$ (this follows from the fact that U is minimized at 0): denoting by $\tilde{V}(\theta)$ what $V(\theta)$ would be if θ faced challengers with probability $q(\theta')$ rather than $q(\theta)$, we have $V(\theta) - V(\theta') \leq \tilde{V}(\theta) - V(\theta') \leq \Delta$ and the same argument applies. $\square$

Proof of Proposition 5. The proof proceeds as follows. First, we will argue that any SSMPE must have the properties described in the Proposition and be of type 1, 2, 3 or 4. We will then show that the equilibrium behavior and value functions are

[29] An analogous argument can be written if the minimum is not attained, by choosing values of θ such that $U(\theta)$ is close to the infimum of U.

uniquely determined given values of θ_0, θ_1, $q(0)$ and q_0 (but only some such choices will in principle lead to an equilibrium), and calculate the value functions. Finally we will show how to pin down θ_0, θ_1, $q(0)$ and q_0.

Characterization

By Proposition 4, $U(\theta)$ is nondecreasing. Combining this, Lemma 1, and the assumption that ties are broken randomly ($r(\theta, \theta') = \frac{1}{2}$ if $U(\theta) = U(\theta')$), we conclude that $T(\theta)$ is weakly decreasing in θ, and constant exactly where $U(\theta)$ is constant (i.e., if $U(\theta) = U(\theta')$ then $T(\theta) = T(\theta')$, and if $U(\theta) < U(\theta')$ then $T(\theta) > T(\theta')$—note that $T(\theta) = T(\theta') = 0$ is not possible because $T(\theta) > 0$ if $\theta < 1$).

Let $A = \{\theta \in [0, 1] : T(\theta) = c\}$. By the above argument $U(\theta)$ must be constant across all $\theta \in A$. Because T is nonincreasing, if A is nonempty, it must be an interval.

The analog of Equation 4 with strategic entry (and under stationary limits) is

$$\begin{aligned} V(\theta) = &\mu \left[q(\theta) E \left[\min(U(\theta), U(\theta'))|\theta' \sim F\right] + (1 - q(\theta))U(0)\right] \\ &+(1 - \mu) \left[q(\theta) E \left[\max(U(\theta), U(\theta'))|\theta' \sim F\right] + (1 - q(\theta))U(\theta)\right]. \end{aligned} \tag{7}$$

Note that $U(\theta)$ cannot be constant a.e. (If it were, $V(\theta) \equiv V(0)$ would also be constant, then $U(\theta) = \theta + \delta p V(0) + \delta(1 - p)V$ would be strictly increasing.) Then $V(\theta)$ is a strictly increasing function of $q(\theta)$ (in the sense of solving for $U(\theta)$ and $V(\theta)$ by applying the Contraction Mapping Theorem to Equations 3 and 7, taking $q(\theta)$, V and $U(\theta')$ for $\theta' \neq \theta$ as fixed). It follows that $q(\theta)$ cannot be discontinously decreasing, i.e., we cannot have $\lim_{\theta \nearrow \theta_0} q(\theta) > \lim_{\theta \searrow \theta_0} q(\theta)$, because $V(\theta)$ would then have a discontinuous drop at θ_0, and so would $U(\theta)$, contradicting Proposition 4. Note also that $U(\theta)$ cannot increase discontinuously unless $q(\theta)$ increases discontinuously at the same point. But this is impossible: if $\theta < \theta'$ and $U(\theta) < U(\theta')$ then, by Lemma 1, either $T(\theta) > c$, so $q(\theta) = 1$, or $T(\theta') < c$, so $q(\theta') = 0$—either way $q(\theta) \geq q(\theta')$. Hence U is continuous.

There are then three possible cases. First, $T(\theta) > c$ for all $\theta \in [0, 1)$, in which case $q(\theta) = 1$ for all θ. This case is incompatible with equilibrium because it implies $U(\theta)$ is strictly increasing everywhere by Proposition 2, but this means that incumbents with θ near 1 are almost unbeatable, i.e., $T(\theta) \xrightarrow[\theta \to 1]{} 0$, a contradiction. Second, $T(\theta) < c$ for all $\theta \in (0, 1]$, in which case $q(\theta) = 0$ for all θ—a type 4 equilibrium. Third, $T(\theta) = c$ for some $\theta \in (0, 1)$ or $T(\theta) > c > T(\theta')$ for some $\theta < \theta'$. In this case A must be an interval with positive measure, as otherwise $q(\theta)$ would drop discontinuously from 1 to 0 where $T(\theta)$ crosses c (because we can only have $q(\theta) \in (0,1)$ if $T(\theta) = c$). We then define $\theta_0 = \inf A$ and $\theta_1 = \sup A$. We say the equilibrium is type 1 if $\theta_1 = 1$, type 3 if $\theta_0 = 0$, and type 2 otherwise (as noted before, U cannot be constant everywhere so we cannot have $\theta_0 = 0$ and $\theta_1 = 1$).

For $\theta < \theta_0$, we can prove U, V are increasing as well as show that $U'(\theta) = \frac{1}{1-\delta p[\mu+(1-2\mu)F(\theta)]}$ and $V'(\theta) = \frac{\mu+(1-2\mu)F(\theta)}{1-\delta p[\mu+(1-2\mu)F(\theta)]}$ in the same way as in part (iii) of Proposition 2.

For $\theta \in [\theta_0, \theta_1]$, we know $U(\theta)$ is constant, i.e., $U'(\theta) = 0$, which implies, because $U(\theta) = \theta + \delta p V(\theta) + \delta(1-p)V$, that $V'(\theta) = \frac{-1}{\delta p}$. Plug in $U(\theta) = U(\theta_0)$ and $V(\theta) = V(\theta_0) - \frac{\theta-\theta_0}{\delta p}$ into Equation 7 to obtain

$$\begin{aligned} V(\theta_0) - \frac{\theta-\theta_0}{\delta p} &= \mu\left[q(\theta)E\left[\min(U(\theta_0), U(\theta'))|\theta' \sim F\right] + (1-q(\theta))U(0)\right] \\ &\quad +(1-\mu)\left[q(\theta)E\left[\max(U(\theta_0), U(\theta'))|\theta' \sim F\right] + (1-q(\theta))U(\theta_0)\right] \\ \Longrightarrow -\frac{1}{\delta p} &= q'(\theta)\left[\mu\left(E\left[\min(U(\theta_0), U(\theta'))|\theta' \sim F\right] - U(0)\right) + \right. \\ &\quad \left. + (1-\mu)\left(E\left[\max(U(\theta_0), U(\theta'))|\theta' \sim F\right] - U(\theta_0)\right)\right]. \qquad (8) \end{aligned}$$

It follows that $q(\theta)$ decreases linearly within $[\theta_0, \theta_1]$.

Finally, for $\theta > \theta_1$, $q(\theta) = 0$, so $V(\theta) = \mu U(0) + (1-\mu)U(\theta)$. Combining this with Equation 3, we find $U(\theta) = \theta + \delta p \mu U(0) + \delta p(1-\mu)U(\theta) + \delta(1-p)V$, or $U(\theta) = \frac{\theta+\delta p\mu U(0)+\delta(1-p)V}{1-\delta p(1-\mu)}$, and $V(\theta) = \frac{(1-\mu)\theta+\mu U(0)+\delta(1-p)(1-\mu)V}{1-\delta p(1-\mu)}$. In particular $U'(\theta) = \frac{1}{1-\delta p(1-\mu)}$, $V'(\theta) = \frac{1-\mu}{1-\delta p(1-\mu)}$.

Calculating the value functions

We will now derive explicit expressions for $U(\theta)$, $V(\theta)$ and V, taking θ_0, θ_1, $q(0)$ and q_0 as given.

Let $\tilde{U}(\theta) = U(\theta) - U(0)$. Because U is continuous and we have expressions for $U'(\theta)$ for $\theta < \theta_0$, $\theta \in (\theta_0, \theta_1)$ and $\theta > \theta_1$, we can calculate $\tilde{U}(\theta)$ as $\int_0^\theta U'(\theta')d\theta'$.

Let us then calculate $U(0)$. By Equation 3, $U(0) = \delta p V(0) + \delta(1-p)V$. By Equation 7, $V(0) = (\mu + (1-\mu)(1-q(0)))U(0) + (1-\mu)q(0)E(U(\theta)|\theta \sim F) = U(0) + (1-\mu)q(0)E(\tilde{U}(\theta)|\theta \sim F)$. Substituting and rearranging we find

$$U(0) = \frac{\delta p q(0)(1-\mu)E(\tilde{U}(\theta)|\theta \sim F) + \delta(1-p)V}{1-\delta p}.$$

Next, we calculate V as follows: $V = q_0^2 A + 2q_0(1-q_0)B + (1-q_0)^2 C$, where

$$\begin{aligned} A &= \mu E\left[\min(U(\theta), U(\theta'))|\theta, \theta' \sim F\right] + (1-\mu)E\left[\max(U(\theta), U(\theta'))|\theta, \theta' \sim F\right] = \\ &= (2\mu - 1)E\left[\min(U(\theta), U(\theta'))|\theta, \theta' \sim F\right] + 2(1-\mu)E\left[U(\theta)|\theta \sim F\right] \\ B &= \mu U(0) + (1-\mu)E\left[U(\theta)|\theta \sim F\right], \quad C = U(0) \end{aligned}$$

Equivalently we can write $V = U(0) + q_0^2 \tilde{A} + 2q_0(1-q_0)\tilde{B}$, where

$$\begin{aligned} \tilde{A} &= (2\mu - 1)E\left[\min(\tilde{U}(\theta), \tilde{U}(\theta'))|\theta, \theta' \sim F\right] + 2(1-\mu)E\left[\tilde{U}(\theta)|\theta \sim F\right] \\ \tilde{B} &= (1-\mu)E\left[\tilde{U}(\theta)|\theta \sim F\right] \end{aligned}$$

Plugging in $\tilde{A}$, $\tilde{B}$ and $U(0)$ and rearranging, we find that

$$\frac{1-\delta}{1-\delta p}V = q_0^2(2\mu - 1)E\left(\min(\tilde{U}(\theta), \tilde{U}(\theta'))\right) + \left(\frac{\delta p(1-\mu)q(0)}{1-\delta p} + 2q_0(1-\mu)\right)E\left(\tilde{U}(\theta)\right).$$

Now recall that $E\left(\min(\tilde{U}(\theta), \tilde{U}(\theta'))|\theta, \theta' \sim F\right) = E\left(\tilde{U}(\theta)|\theta \sim 1-(1-F)^2\right)$. In addition, for any cdf G with density g s.t. $G(0) = 0$ and $G(1) = 1$,

$$\begin{aligned} E\left(\tilde{U}(\theta)|\theta \sim G\right) &= \int_0^1 \tilde{U}(\theta)g(\theta)d\theta = \tilde{U}(1) - \int_0^1 \tilde{U}'(\theta)G(\theta)d\theta \\ &= \int_0^1 \tilde{U}'(\theta)d\theta - \int_0^1 \tilde{U}'(\theta)G(\theta)d\theta = \int_0^1 \tilde{U}'(\theta)(1-G(\theta))d\theta. \end{aligned}$$

Applying this result,

$$\begin{aligned} E\left(\min(\tilde{U}(\theta), \tilde{U}(\theta'))|\theta, \theta' \sim F\right) &= \int_0^{\theta_0} \frac{(1-F(\theta))^2}{1-\delta p\left[\mu + (1-2\mu)F(\theta)\right]}d\theta + \int_{\theta_1}^1 \frac{(1-F(\theta))^2}{1-\delta p(1-\mu)}d\theta \\ E\left(\tilde{U}(\theta)|\theta \sim F\right) &= \int_0^{\theta_0} \frac{1-F(\theta)}{1-\delta p\left[\mu + (1-2\mu)F(\theta)\right]}d\theta + \int_{\theta_1}^1 \frac{1-F(\theta)}{1-\delta p(1-\mu)}d\theta. \end{aligned}$$

We now have expressions for $U(\theta)$, $V(\theta)$ and V in terms of the parameters as well as θ_0, θ_1, $q(0)$ and q_0.

Pinning down θ_1

We can rewrite Equation 8 as

$$q'(\theta) = -\frac{1}{\delta p\Big(\mu E\Big[\min(\tilde{U}(\theta_0),\tilde{U}(\theta'))|\theta'\sim F\Big]+(1-\mu)\Big(E\Big[\max(\tilde{U}(\theta_0),\tilde{U}(\theta'))|\theta'\sim F\Big]-\tilde{U}(\theta_0)\Big)\Big)},\text{ where}$$

$$\begin{aligned} E\left[\min(\tilde{U}(\theta_0),\tilde{U}(\theta'))|\theta'\sim F\right] &= \int_0^{\theta_0}\frac{1-F(\theta)}{1-\delta p\left[\mu+(1-2\mu)F(\theta)\right]}d\theta \\ E\left[\max(\tilde{U}(\theta_0),\tilde{U}(\theta'))|\theta'\sim F\right]-\tilde{U}(\theta_0) &= \int_{\theta_1}^{1}\frac{1-F(\theta)}{1-\delta p(1-\mu)}d\theta. \end{aligned}$$

We have thus expressed q' as a function of θ_0, θ_1 and parameters. In particular, $|q'|$ is weakly increasing in θ_1. With this observation we can uniquely determine θ_1 as a function of θ_0 and $q(0)$. To see how, suppose that $\theta_1 < 1$, so $q(\theta_1) = 0$. Then we must have $|q'|(\theta_1 - \theta_0) = 1$ if $\theta_0 > 0$, or $|q'|\theta_1 = q(0)$ if $\theta_0 = 0$. Whichever case applies, the left-hand side of the equation is increasing in θ_1, so there is a unique value of θ_1 that solves it given θ_0 and $q(0)$. (Unless the left-hand side is smaller the right-hand side even for $\theta_1 = 1$, in which case this is the solution.)

Pinning down q_0

To determine q_0, consider the incentive to run in an open election. If the other party's potential challenger is running with probability q, it is weakly optimal to run iff $qT_1+(1-q)T_2-c \geq 0$, where T_1 is the expected reward from running when the other challenger also runs, and T_2 is the expected reward from running against a challenger who doesn't. Note that, having assumed values of θ_0 and $q(0)$ (which pin down θ_1, hence $q(\theta)$ and $\tilde{U}$), T_1 and T_2 are uniquely determined. By Lemma 1, $T_1 < T_2$, so a challenger's incentive to run is decreasing in the other challenger's probability of running. Since in equilibrium both must run with probability q_0, there are three possible cases. Either $T_1 \geq c$ and $q_0 = 1$, or $T_2 \leq c$ and $q_0 = 0$, or $T_1 < c < T_2$ and q_0 is uniquely determined by the equation $q_0T_1 + (1 - q_0)T_2 = c$, i.e., $q_0 = \frac{T_2-c}{T_2-T_1}$.

Pinning down θ_0 and $q(0)$

There is really only one degree of freedom in choosing θ_0 and $q(0)$, rather than two: either $\theta_0 > 0$ and $q(0) = 1$ or $\theta_0 = 0$ and $q(0) \in [0, 1]$.

Given values of θ_0 and $q(0)$, we have shown that θ_1, q_0, $U(\theta)$, $V(\theta)$, V, and $T(\theta)$ are uniquely determined. Our construction guarantees that the objects thus found are compatible with equilibrium, with one exception: we have not yet checked the challenger's entry decision. It is necessary and sufficient that $\overline{T} = c$, where $\overline{T} = T(\theta)$ for $\theta \in [\theta_0, \theta_1]$. Under the conditions provided in the Proposition, $\overline{T}$ is a decreasing function of θ_0—an intuitive fact since θ_0 parameterizes the level of competition, which should lower the expected future rents. Hence, there is a unique value of θ_0 such that $\overline{T}(\theta_0) = c$, and the SSMPE is unique.

Parameter conditions are required to guarantee equilibrium uniqueness because increasing θ_0 has three effects on $\overline{T}$, one of which has the "wrong" sign:

1. It increases future competition (by shifting the function $q(\theta)$ upwards), which lowers expected tenure and rents, reducing $\overline{T}$.

2. It shifts the set $A = (\theta_0, \theta_1)$ upwards, making incumbents with $\theta \in A$ electorally stronger. This reduces $\overline{T}$ since challengers have a lower chance of winning their first election against someone with $\theta \in A$.

3. Yet, conditional on winning the first election, these challengers-turned-incumbents will be electorally stronger themselves by the same logic, which increases $\overline{T}$.

$\overline{T}$ is decreasing in θ_0 unless the third effect dominates. The conditions for equilibrium uniqueness are checked in Appendix B.

□

Proof of Proposition 6. $T_k(\theta)$ is a decreasing function of $U_k(\theta)$ (Lemma 1). Then there is a threshold U^* such that (abusing notation) $T(U^*) = c$, so that $q_k(\theta) = 1$ for $U_k(\theta) < U^*$ and $q_k(\theta) = 0$ for $U_k(\theta) > U^*$.

Given U^*, V and a candidate function $U_0(\theta)$, we can solve for $U_k(\theta)$ with a form of backward induction. First, $U_{m-1}(\theta) = \theta + \delta V$, which is linear and increasing. If $\theta_{1(m-2)} + \delta V = U^*$ for some $\theta_{1(m-2)}$, then $U_{m-2}(\theta) = \theta + \delta(\mu E[\min(U_{m-1}(\theta), U_0(\theta'))] + (1-\mu)E[\max(U_{m-1}(\theta), U_0(\theta'))])$, $q_{m-2}(\theta) = 1$ for $\theta < \theta_{1(m-2)}$ and $U_{m-2}(\theta) = \theta + \delta(\mu \min(U_{m-1}(\theta), U_0(0)) + \mu \max(U_{m-1}(\theta), U_0(0)))$, $q_{m-2}(\theta) = 0$ for $\theta > \theta_{1(m-2)}$. We can proceed like this for $k = m-3, \ldots, 0$. This argument doubles as an algorithm for equilibrium construction: pick candidate values of U^*, V and $U_0(\theta)$ and solve for an equilibrium as above. The resulting strategy profile is an equilibrium iff the generated U^*, V and $U_0(\theta)$ match the conjectures.

This construction also yields the cutoffs $(A_k)_k$. To show that $l_k \leq 2^{m-k-1} - 1$ we argue as follows. Let $h(U)$ be a function given by $h(U) = \mu E[\min(U, U_0(\theta'))|\theta' \sim$

$F] + (1-\mu)E[\max(U, U_0(\theta'))|\theta' \sim F]$ if $U < U^*$ and $h(U) = \mu \min(U, U_0(0)) + (1-\mu)\max(U, U_0(0))$ if $U > U^*$. Define $h_\theta(U) = \theta + \delta h(U)$. Then $U_{k-1}(\theta) = h_\theta(U_k(\theta))$. Clearly $h_\theta(U)$ is smoothly increasing in θ, hence $U_k(\theta) = h_\theta^{m-1-k}(U_{m-1}(\theta))$ is smoothly increasing in θ, except at points of discontinuity of $U_{k'}(\theta)$ ($k' > k$) and points where $U_{k+1}(\theta) = U^*$. If U_{k+1} has l_{k+1} jumps, it is smoothly increasing in $l_{k+1}+1$ intervals, in each of which it can cross U^* at most once. Hence U_k has at most $2l_{k+1}+1$ jumps, i.e., $l_k \leq 2l_{k+1}+1$. This plus the fact that $l_{m-2} \leq 1$ delivers the result. □

Proof of Prpoosition 7. Note that if $c < c_0 = \frac{b}{2}$ then $q_0 = 1$. The reason is that running in an open election increases the chance of winning by at least $\frac{1}{2}$, which is worth at least $\frac{b}{2}$. Let $V((p_k)_k, c)$ be the value of V under term limits $(p_k)_k$ and a cost of entry c. By an abuse of notation $V((p, p, \ldots), c) = V(p, c)$. Then, for all $c \in (0, c_0)$, $V(0, c) = V(0, 0)$. The reason is that, when $p = 0$, every election is open, so only q_0 matters for welfare. However, for any other $(p_k)_k$, $V((p_k)_k, c) \leq V((p_k)_k, 0)$ since $c > 0$ results in a weakly lower $q(\theta)$ relative to $q(\theta) \equiv 1$. Then, whenever $V(0,0) \geq V((p_k)_k, 0)$ we have $V(0, c) \geq V((p_k)_k, c)$ for all $c \in (0, c_0)$.

□

Proof of Proposition 8. Analogous to Proposition 5. □

B Supplementary Appendix (for Online Publication)

Proof of Lemma 1. Consider a challenger i who first runs in period t against an incumbent of type (θ, k). Let $R(\theta')$ be i's expected lifetime rents from office, conditional on winning in period t and on her ability being θ'. Let $\gamma Q(\theta')$ be i's expected lifetime policy payoffs **excluding period** t, again conditional on winning in period t and her ability being θ'. Let $\gamma S_k(\theta, \theta')$ be i's policy payoff in period t, conditional on her ability being θ' and the incumbent being type (θ, k). (Note that $R(\theta')$, $Q(\theta')$ are independent of θ and k, and R, Q and S are *not* functions of γ.) Then

$$T_k(\theta) = \int_0^1 \left[R(\theta') + \gamma Q(\theta') + \gamma S_k(\theta, \theta')\right] r_k(\theta, \theta') f(\theta') d\theta'.$$

By Proposition 1, if the challenger wins and her ability is θ', then with probability $1-\mu$ she is unbiased, her policy is 0, and her policy payoff is 0; with probability μ she is biased, her policy is $\pm\sqrt{\frac{U_0(\theta')-U_k(\theta)}{\lambda}}$, and her policy payoff is $-\left(I - \sqrt{\frac{U_0(\theta')-U_k(\theta)}{\lambda}}\right)^2$. In other words, $S_k(\theta, \theta') = -\mu \left(I - \sqrt{\frac{U_0(\theta')-U_k(\theta)}{\lambda}}\right)^2$, which is a strictly decreasing function of $U_k(\theta)$. Furthermore, $r_k(\theta, \theta')$ is weakly decreasing as a function of $U_k(\theta)$ for each θ': if $U_k(\theta) < U_{\tilde{k}}(\tilde{\theta})$, then either $U_k(\theta) < U_0(\theta')$, implying $r_k(\theta, \theta') = 1 \geq r_{\tilde{k}}(\tilde{\theta}, \theta')$, or $U_0(\theta') < U_{\tilde{k}}(\tilde{\theta})$, implying $r_k(\theta, \theta') \geq r_{\tilde{k}}(\tilde{\theta}, \theta') = 0$. The result follows. □

Proof of Proposition 5–Pinning down θ_0. Under stationary limits, the expressions for R and Q simplify to

$$\begin{aligned} R(\theta) &= \frac{b}{1 - \delta p(1 - q(\theta)\kappa(\theta))} = \frac{b}{1 - \delta p + \delta p q(\theta)\kappa(\theta)} \\ Q(\theta) &= \left[q(\theta)y_1 + (1 - q(\theta))y_0\right] \frac{\delta p}{1 - \delta p + \delta p q(\theta)\kappa(\theta)}, \end{aligned}$$

where $\kappa(\theta) = \int_0^1 r(\theta, \theta') f(\theta') d\theta'$ is the probability that an incumbent of ability θ loses an election, conditional on the challenger running; and y_1, y_0 are the expected flow policy payoffs of an incumbent of ability θ if the challenger runs or does not run, respectively. Remember also that

$$\overline{T}_{\theta_0} = \int_0^1 \left(R(\theta) + \gamma Q(\theta) + \gamma S(\theta_0, \theta)\right) r(\theta_0, \theta) f(\theta)\, d\theta.$$

Suppose first that the equilibrium is of type 2, and let $\theta_1 = \theta_1(\theta_0)$. Then $r(\theta_0, \theta) = 0$ for $\theta < \theta_0$, $r(\theta_0, \theta) = \frac{1}{2}$ for $\theta \in [\theta_0, \theta_1]$ and $r(\theta_0, \theta) = 1$ for $\theta > \theta_1$:

$$\overline{T}_{\theta_0} = \frac{1}{2}\int_{\theta_0}^{\theta_1} \left(R(\theta) + \gamma Q(\theta) + \gamma S(\theta_0, \theta)\right) f(\theta)\, d\theta + \int_{\theta_1}^{1} \left(R(\theta) + \gamma Q(\theta) + \gamma S(\theta_0, \theta)\right) f(\theta)\, d\theta.$$

Letting $R_* = \frac{\partial R}{\partial \theta_0}$ and so on, we then want to show that $\frac{\partial \overline{T}_{\theta_0}}{\partial \theta_0} < 0$ for all θ_0, where

$$\begin{aligned}\frac{\partial \overline{T}_{\theta_0}}{\partial \theta_0} &= \frac{1}{2}\int_{\theta_0}^{\theta_1} \left(R_*(\theta) + \gamma Q_*(\theta) + \gamma S_*(\theta_0, \theta)\right) f(\theta)\, d\theta + \int_{\theta_1}^{1} \left(R_*(\theta) + \gamma Q_*(\theta) + \gamma S_*(\theta_0, \theta)\right) f(\theta)\, d\theta \\ &\quad - \frac{1}{2}(R(\theta_0) + \gamma Q(\theta_0) + \gamma S(\theta_0, \theta_0)) f(\theta_0) - \frac{1}{2}\theta_1'(\theta_0)(R(\theta_1) + \gamma Q(\theta_1) + \gamma S(\theta_0, \theta_1)) f(\theta_1).\end{aligned}$$

Note that $R_*(\theta) = 0$ for $\theta > \theta_1$ (because $q(\theta)\kappa(\theta) \equiv 0$), and $S(\theta_0, \theta) = S_*(\theta_0, \theta) = 0$ for $\theta \in [\theta_0, \theta_1]$. Then we need to show

$$\begin{aligned}&\frac{1}{2}\int_{\theta_0}^{\theta_1} \left(R_*(\theta) + \gamma Q_*(\theta)\right) f(\theta)\, d\theta + \int_{\theta_1}^{1} \left(\gamma Q_*(\theta) + \gamma S_*(\theta_0, \theta)\right) f(\theta)\, d\theta \\ &- \frac{1}{2}(R(\theta_0) + \gamma Q(\theta_0)) f(\theta_0) - \frac{1}{2}\theta_1'(\theta_0)(R(\theta_1) + \gamma Q(\theta_1)) f(\theta_1) < 0\end{aligned}$$

Because we want to show this holds for γ low enough, it is necessary and sufficient to prove that

$$\int_{\theta_0}^{\theta_1} R_*(\theta) f(\theta)\, d\theta < R(\theta_0) f(\theta_0) + R(\theta_1) f(\theta_1)\theta_1'(\theta_0) \tag{9}$$

and that $Q_*(\theta)$, $Q(\theta)$, and $S_*(\theta_0, \theta)$ $(\theta > \theta_1)$ are bounded.[30] Before proceeding further, note that R_*, Q_* and S_* (hence also q_* and κ_*) must be well defined for our approach to be valid. This boils down to showing that $\theta_1'(\theta_0)$ exists, which follows from applying the Implicit Function Theorem to the characterization of θ_1 in Lemma 3.

We will first deal with office rents. We can calculate

$$R_*(\theta) = \frac{b\delta p q(\theta)\kappa(\theta)}{(1 - \delta p + \delta p q(\theta)\kappa(\theta))^2}\left(-\frac{q_*(\theta)}{q(\theta)} - \frac{\kappa_*(\theta)}{\kappa(\theta)}\right).$$

Here $\kappa(\theta) = 1 - \frac{F(\theta_0)+F(\theta_1)}{2}$, so $\kappa_*(\theta) = -\frac{f(\theta_0)+f(\theta_1)\theta_1'(\theta_0)}{2}$, and $q(\theta) = \frac{\theta_1-\theta}{\theta_1-\theta_0}$, so $q_*(\theta) =$

[30] Because both sides of (9) are continuous in θ_0, if the inequality holds strictly for all θ_0, the difference between the two sides is bounded away from zero.

$\frac{\theta_1'(\theta_0)(\theta-\theta_0)+\theta_1-\theta}{(\theta_1-\theta_0)^2}$. A digression here will be necessary. Using our characterization of q' and θ_1 (Proposition 5—pinning down θ_1), we can show that $\theta_1-\theta_0$ is bounded away from zero and θ_1' is bounded and bounded away from zero:

Lemma 3. *There are m, m', $M>0$ dependent only on μ, δ, p and F such that $\theta_1(\theta_0)-\theta_0\geq m'$ and $\theta_1'(\theta_0)\in[m,M]$.*

Proof. Note that if $\theta_1(\theta_{0k})-\theta_{0k}\xrightarrow[k\to\infty]{}0$ for some sequence $(\theta_{0k})_k$, then in the limit we would have $|q'|\leq\frac{1}{\delta p\min(\mu,1-\mu)\int_0^1\min\left(\frac{1-F(\theta)}{1-\delta p[\mu+(1-2\mu)F(\theta)]},\frac{1-F(\theta)}{1-\delta p(1-\mu)}\right)d\theta}<\infty$, so $q'(\theta_1-\theta_0)\to 0$, a contradiction. If $1\geq\theta_1-\theta_0\geq m'$, then $1\leq q'\leq\frac{1}{m'}$. θ_1' must solve $q'(\theta_1'-1)+\left(\frac{\partial q'}{\partial\theta_1}\theta_1'+\frac{\partial q'}{\partial\theta_0}\right)(\theta_1-\theta_0)=0$, or $\theta_1'=\frac{q'-\frac{\partial q'}{\partial\theta_0}}{q'+\frac{\partial q'}{\partial\theta_1}}$. Here $-\frac{\partial q'}{\partial\theta_0}=q'^2\frac{\delta p\mu(1-F(\theta_0))}{1-\delta p[\mu+(1-2\mu)F(\theta_0)]}\leq\frac{\delta p\mu}{(1-\delta p)m'^2}$ and $\frac{\partial q'}{\partial\theta_1}=q'^2\frac{\delta p(1-\mu)(1-F(\theta_1))}{1-\delta p+\delta p\mu}\leq\frac{\delta p(1-\mu)}{(1-\delta p)m'^2}$. This yields the result. $\square$

Using Lemma 3 and previous results, and denoting $\underline{m}=\min(m,1)$,

$$
\begin{aligned}
-\frac{q_*(\theta)}{q(\theta)}&=-\frac{1}{q(\theta)}\frac{\theta_1'(\theta_0)(\theta-\theta_0)+\theta_1-\theta}{(\theta_1-\theta_0)^2}\leq-\frac{1}{q(\theta)}\frac{\underline{m}(\theta-\theta_0)+(\theta_1-\theta)}{\theta_1-\theta_0}=\\
&=-\frac{\underline{m}}{q(\theta)(\theta_1-\theta_0)}-\frac{1-\underline{m}}{\theta_1-\theta_0}\leq-\frac{1}{1-\theta_0}\left(\frac{\underline{m}}{q(\theta)}+1-\underline{m}\right)\\
-\frac{\kappa_*(\theta)}{\kappa(\theta)}&=\frac{f(\theta_0)+f(\theta_1)\theta_1'(\theta_0)}{2-F(\theta_0)-F(\theta_1)}\leq\frac{f(\theta_0)+f(\theta_1)\theta_1'(\theta_0)}{1-F(\theta_0)}.
\end{aligned}
$$

Then we can deal with the terms involving $f(\theta_1)$ as follows:

$$
\int_{\theta_0}^{\theta_1}\frac{b\delta pq(\theta)\kappa(\theta)}{(1-\delta p+\delta pq(\theta)\kappa(\theta))^2}\frac{f(\theta_1)\theta_1'(\theta_0)}{(1-F(\theta_0))}f(\theta)d\theta<R(\theta_1)f(\theta_1)\theta_1'(\theta_0),
$$

because $\frac{\delta pq(\theta)\kappa(\theta)}{(1-\delta p+\delta pq(\theta)\kappa(\theta))^2}<\frac{1}{1-\delta p}$ and $\int_{\theta_0}^{\theta_1}f(\theta)d\theta\leq 1-F(\theta_0)$. So it is enough to show

$$
\int_{\theta_0}^{\theta_1}\frac{b\delta pq(\theta)\kappa(\theta)}{(1-\delta p+\delta pq(\theta)\kappa(\theta))^2}\left(-\frac{\frac{\underline{m}}{q(\theta)}+1-\underline{m}}{1-\theta_0}+\frac{f(\theta_0)}{1-F(\theta_0)}\right)f(\theta)d\theta<R(\theta_0)f(\theta_0).
$$

Using that $\frac{f(\theta_0)}{1-F(\theta_0)}\leq\frac{\phi}{1-\theta_0}$, it is enough to show that for any $0\leq q\leq 1$

$$\left(\frac{b\delta pq\kappa}{(1-\delta p+\delta pq\kappa)^2}\left(1-\frac{\underline{m}}{\phi q}-\frac{1-\underline{m}}{\phi}\right)\frac{f(\theta_0)}{1-F(\theta_0)}\right)(F(\theta_1)-F(\theta_0))<\frac{bf(\theta_0)}{1-\delta p+\delta p\kappa}$$
$$\frac{\delta pq\kappa}{(1-\delta p+\delta pq\kappa)^2}\left(1-\frac{\underline{m}}{\phi q}-\frac{1-\underline{m}}{\phi}\right)<\frac{1}{1-\delta p+\delta p\kappa}$$

The left-hand side is single-peaked in q with a maximum at $q^*=\frac{1-\delta p}{\delta pk}+\frac{2\underline{m}}{\phi-1+\underline{m}}$. If this q^* is greater than 1, then we need

$$\frac{\delta p\kappa}{(1-\delta p+\delta p\kappa)^2}\left(1-\frac{1}{\phi}\right)<\frac{1}{1-\delta p+\delta p\kappa},$$

which always holds. If $0<q^*<1$, then the maximized value of the left-hand side is $\frac{1}{\frac{4\phi}{\phi-1+\underline{m}}(1-\delta p)+\frac{4\underline{m}\phi}{(\phi-1+\underline{m})^2}\delta p\kappa}$. Since $\underline{m}\leq 1$, $\frac{4\phi}{\phi-1+\underline{m}}\geq 4>1$, so the required inequality is guaranteed to hold if $\frac{4\underline{m}\phi}{(\phi-1+\underline{m})^2}$ is at least 1. This expression is decreasing in ϕ (again given $\underline{m}\leq 1$) and equals $\frac{4}{\underline{m}}>1$ if $\phi=1$, so there is $\phi^*(\underline{m})>1$ such that the inequality holds whenever $\phi\leq\phi^*(\underline{m})$.

We now turn to policy payoffs. For $\theta\in[\theta_0,\theta_1]$,

$$Q(\theta)=[q(\theta)y_1+(1-q(\theta))y_0]\frac{\delta p}{1-\delta p+\delta pq(\theta)\kappa(\theta)}$$
$$\Longrightarrow Q_*(\theta)=-\frac{q(\theta)y_1+(1-q(\theta))y_0}{(1-\delta p+\delta pq(\theta)\kappa(\theta))^2}\delta^2p^2q(\theta)\kappa(\theta)\left(\frac{q_*(\theta)}{q(\theta)}+\frac{\kappa_*(\theta)}{\kappa(\theta)}\right)$$
$$-\delta p\frac{q_*(\theta)(y_0-y_1)}{1-\delta p+\delta pq(\theta)\kappa(\theta)}+\delta p\frac{q(\theta)y_{1*}+(1-q(\theta))y_{0*}}{1-\delta p+\delta pq(\theta)\kappa(\theta)}.$$

f is bounded by assumption and $q,\kappa\leq 1$. Also $|Q(\theta)|,|y_0|,|y_1|\leq\frac{I^2}{1-\delta p}$. It remains to bound y_{0*} and y_{1*}. Using that $y_0=S(0,\theta)$, $y_1=\int_0^1 S(\theta',\theta)f(\theta')d\theta$, and $S(\theta',\theta)=\mu\left(-\frac{U(\theta)-U(\theta')}{\lambda}+2\sqrt{\frac{U(\theta)-U(\theta')}{\lambda}}I-I^2\right)$ for any $\theta'\leq\theta$ (see Lemma 1), we obtain:

$$y_0=\mu\left(-I^2+2I\sqrt{\frac{\tilde{U}(\theta_0)}{\lambda}}-\frac{\tilde{U}(\theta_0)}{\lambda}\right),\quad y_{0*}=\mu U'(\theta_0)\left[I\sqrt{\frac{1}{\tilde{U}(\theta_0)\lambda}}-\frac{1}{\lambda}\right],$$

$$y_1 = \mu \int_0^{\theta_0} \left(-I^2 + 2I\sqrt{\frac{\tilde{U}(\theta_0) - \tilde{U}(\theta)}{\lambda}} - \frac{\tilde{U}(\theta_0) - \tilde{U}(\theta)}{\lambda} \right) f(\theta) d\theta,$$

$$y_{1*} = \mu U'(\theta_0) \int_0^{\theta_0} \left(I\sqrt{\frac{1}{(\tilde{U}(\theta_0) - \tilde{U}(\theta))\lambda}} - \frac{1}{\lambda} \right) f(\theta) d\theta.$$

Now, using that $1 \leq U'(\theta) \leq \frac{1}{1-\delta p}$ for $\theta < \theta_0$, and denoting $\max f = \overline{f}$,

$$-\frac{\mu}{\lambda(1-\delta p)} \leq y_{1*} \leq \frac{\mu}{1-\delta p} \int_0^{\theta_0} I\sqrt{\frac{1}{(\theta_0 - \theta)\lambda}} \overline{f} d\theta = \frac{\mu}{1-\delta p} \frac{2I\overline{f}\sqrt{\theta_0}}{\sqrt{\lambda}} \leq \frac{\mu}{1-\delta p} \frac{2I\overline{f}}{\sqrt{\lambda}}$$

$$-\frac{\mu}{\lambda(1-\delta p)} \leq y_{0*} \leq \frac{\mu}{1-\delta p} \frac{I}{\sqrt{\theta_0}\sqrt{\lambda}}.$$

$Q_*(\theta)$ for $\theta > \theta_1$ and $S_*(\theta_0, \theta)$ for $\theta > \theta_1$ can be bounded with similar arguments. All of our bounds are uniform in θ_0 except for the upper bound on y_{0*}, which is proportional to $\frac{1}{\sqrt{\theta_0}}$ and explodes as $\theta_0 \to 0$.

We finish our proof of equilibrium uniqueness in this region with the following argument. If $\gamma = 0$, given values of all other parameters, there is a unique equilibrium whenever $\phi < \phi^*(\underline{m})$. Let θ^* be the value of θ_0 in this equilibrium. If $\theta^* > 0$, the *marginal* policy payoffs that show up in $\frac{\partial \overline{T}}{\partial \theta_0}$ are bounded in a neighborhood of θ^*, and the *total* policy payoffs in $\overline{T}(\theta)$ are bounded everywhere (i.e., $\overline{T}$ may be nonmonotonic near 0, but this is far from θ^*, where $\overline{T}$ crosses c). If $\theta^* = 0$, then $\overline{T}(\theta^*) < c$ for any $\gamma > 0$ (because policy payoffs are negative), so the equilibrium is type 3, which does not have these issues.

Next, suppose the equilibrium is type 1. Then

$$\overline{T}_{\theta_0} = \frac{1}{2} \int_{\theta_0}^1 \left(R(\theta) + \gamma Q(\theta) + \gamma S(\theta_0, \theta) \right) f(\theta)\, d\theta$$

$$\frac{\partial \overline{T}_{\theta_0}}{\partial \theta_0} = \frac{1}{2} \int_{\theta_0}^1 \left(R_*(\theta) + \gamma Q_*(\theta) + \gamma S_*(\theta_0, \theta) \right) f(\theta)\, d\theta - \frac{1}{2}(R(\theta_0) + \gamma Q(\theta_0) + \gamma S(\theta_0, \theta_0)) f(\theta_0)$$

Bounding the policy payoffs in this case is not hard (the issues that arise as θ_0

approaches zero do not apply here). We then have to show

$$\int_{\theta_0}^{1} R_*(\theta) f(\theta)\, d\theta < R(\theta_0) f(\theta_0).$$

We now have

$$q_*(\theta) \geq \frac{1-q(1)}{1-\theta_0},\ \kappa(\theta) = \frac{1-F(\theta_0)}{2} \Longrightarrow \kappa_*(\theta) = -\frac{1}{2} f(\theta_0),\ -\frac{\kappa_*(\theta)}{\kappa(\theta)} \leq \frac{f(\theta_0)}{1-F(\theta_0)}.$$

(The bound on $q_*(\theta)$ uses the fact that, when $\theta_1 = 1$, $|q'(\theta)|$ is decreasing in θ_0—see Proposition 5.) Arguing as before, it is enough to show

$$\frac{b\delta pq\kappa}{(1-\delta p+\delta pq\kappa)^2}\left(1-\frac{1-q(1)}{\phi q}\right)\frac{f(\theta_0)}{1-F(\theta_0)}(1-F(\theta_0)) < \frac{bf(\theta_0)}{1-\delta p+\delta p\kappa}$$
$$\Longleftrightarrow \frac{\delta pq\kappa}{(1-\delta p+\delta pq\kappa)^2}\left(1-\frac{1-q(1)}{\phi q}\right) < \frac{1}{1-\delta p+\delta p\kappa}$$

subject to $q \geq q(1)$.

Again $\frac{\delta pq\kappa}{(1-\delta p+\delta pq\kappa)^2}\left(1-\frac{1-q(1)}{\phi q}\right)$ is single peaked in q with a maximum at $q^* = \frac{1-\delta p}{\delta pk} + \frac{2(1-q(1))}{\phi}$. There are three cases. If $q^* > 1$, then we need

$$\frac{\delta p\kappa}{(1-\delta p+\delta p\kappa)^2}\left(1-\frac{1-q(1)}{\phi}\right) < \frac{1}{1-\delta p+\delta p\kappa},$$

which always holds. If $1 > q^* > q(1)$, then $q^* > \frac{\frac{1-\delta p}{\delta pk}+\frac{2}{\phi}}{1+\frac{2}{\phi}} > q(1)$, and

$$\frac{\delta pq^*\kappa}{(1-\delta p+\delta pq^*\kappa)^2}\left(1-\frac{1-q(1)}{\phi}\right) =$$
$$=\frac{1}{4\left(1-\delta p+\frac{\delta p\kappa}{\phi}(1-q(1))\right)} < \frac{1}{4\left(1-\delta p+\frac{\delta p\kappa}{\phi}\left(1-\frac{\frac{1-\delta p}{\delta pk}+\frac{2}{\phi}}{1+\frac{2}{\phi}}\right)\right)} =$$
$$=\frac{1}{4\left((1-\delta p)\left(1-\frac{1}{\phi+2}\right)+\delta p\kappa\frac{1}{\phi+2}\right)}$$

which is always smaller than $\frac{1}{1-\delta p+\delta p\kappa}$ if $\phi < 2$.

Finally, if $q(1) > q^*$, then we need

$$\frac{\delta p q(1)\kappa}{(1-\delta p+\delta p q(1)\kappa)^2}\left(1-\frac{1-q(1)}{\phi q(1)}\right) < \frac{1}{1-\delta p+\delta p\kappa}$$
$$\iff \frac{\delta p q(1)\kappa}{(1-\delta p+\delta p q(1)\kappa)^2}\frac{\phi+1}{\phi}\left(1-\frac{1}{(\phi+1)q(1)}\right) < \frac{1}{1-\delta p+\delta p\kappa}$$

The value of $q(1)$ that maximizes the left-hand side is $\frac{1-\delta p}{\delta p\kappa}+\frac{2}{\phi+1}$, and the maximized value of the left-hand side is $\frac{\phi+1}{\phi}\frac{1}{4(1-\delta p)+\frac{4}{\phi+1}\delta p\kappa}$. This expression is decreasing in ϕ and always less than $\frac{1}{1-\delta p+\delta p\kappa}$ for $\phi = 1$, so there is again a threshold $\phi^* > 1$ such that the inequality holds if $\phi < \phi^*$.

The case of a type 3 equilibrium is the simplest one. The policy payoffs can be handled as before. For office rents, we need to show that

$$\int_0^{\theta_1} R_*(\theta)f(\theta)d\theta < R(\theta_1)f(\theta_1),$$

where $R_*(\theta)$ now represents $\frac{\partial R(\theta)}{\partial\theta_1}$. (We can't use θ_0 as the parameter since it is 0, and θ_1 is more convenient than $q(0)$.) We can, as before, show that $q_*(\theta) > 0$, and $\kappa(\theta) = 1-\frac{F(\theta_1)}{2}$, so $\kappa_*(\theta) = -\frac{f(\theta_1)}{2}$ and $-\frac{\kappa_*(\theta)}{\kappa(\theta)} = \frac{f(\theta_1)}{2-F(\theta_1)} < f(\theta_1)$. Then

$$R_*(\theta) = \frac{b\delta p q(\theta)\kappa(\theta)}{(1-\delta p+\delta p q(\theta)\kappa(\theta))^2}\left(-\frac{q_*(\theta)}{q(\theta)}-\frac{\kappa_*(\theta)}{\kappa(\theta)}\right) < \frac{b}{1-\delta p}f(\theta_1)$$
$$\implies \int_0^{\theta_1} R_*(\theta)f(\theta)d\theta < \frac{b}{1-\delta p}f(\theta_1)F(\theta_1) < \frac{b}{1-\delta p}f(\theta_1) = R(\theta_1)f(\theta_1).$$

□

Proof of Corollary 1. Parts (i) and (ii) are immediate consequences of Proposition 6. For part (iii), note that $U_1(\theta) = \theta+\delta V$ and $U_0(\theta) = \theta+\delta V_1(\theta)$, so $U_1'(\theta) = 1$ and $U_0'(\theta) = 1+\delta V_1'(\theta)$. For $\theta < \theta_0$, $V_1(\theta) = \mu E(\min(U_1(\theta), U_0(\theta'))|\theta' \sim F) + (1-\mu)E(\max(U_1(\theta), U_0(\theta'))|\theta' \sim F)$. $U_1'(\theta) = 1$then implies $V_1'(\theta)$, so $U_0'(\theta) > U_1'(\theta)$. For $\theta > \theta_0$, $V_1(\theta) = \mu\min(U_1(\theta), U_0(0)) + (1-\mu)\max(U_1(\theta), U_0(0))$. $U_1'(\theta) = 1$ again implies $V_1'(\theta) > 0$ and $U_0'(\theta) > U_1'(\theta)$ unless $\mu = 1$, in which case $V_1(\theta) = U_0(0)$ and $U_0'(\theta) = 1 = U_1'(\theta)$.

For part (iv), if $\mu = 1$, we will argue that $U_0(0) < U_1(\theta)$ for all θ. This follows since $U_0(0) = \delta V_1(0) \leq V_1(0) = V_1(0) = E(\min(U_1(0), U_0(\theta')|\theta' \sim F) \leq U_1(0)$, and

U_1 is increasing. (Note that V, U_0, U_1, $V_1 \geq 0$, since electing the weaker candidate always gives a nonnegative flow payoff.) Hence $V_1(\theta) = U_0(0)$ for $\theta > \theta_0$. It also follows that $U_0(0) \leq V$, as $U_0(0) \leq U_1(0) = \delta V$. Hence $U_1(\theta) \geq U_0(\theta)$ for $\theta > \theta_0$, as $V \geq V_1(\theta) = U_0(0)$ for $\theta > \theta_0$. Both inequalities are strict unless $V = 0$, which happens iff $q_0 = 0$. This argument also goes through for μ in a neighborhood of 1.

There are two degenerate cases. If U^* is above $U_1(1)$, there always is competition. This is possible in under classic limits if c is low enough, since in an open election there is always a positive probability of winning, and in a closed election the challenger can always defeat the incumbent with non-negligible probability, since $U_0(1) > U_1(1)$ (see part (iv) of Proposition 2). If U^* is below $U_1(0)$, there never is competition in a closed election. This is possible if c is high enough. $\square$

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