Noisy Screening and Brinkmanship

PRELIMINARY AND INCOMPLETE

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Introduction

In this paper I study a repeated relationship between a proposer (he) and a receiver (she) subject to termination

In each period, proposer proposes a transfer, which we call a demand

Receiver can accept and continue the relationship, or quit and take an outside option (permanent), ending the game

Receiver's outside option is persistent and privately known

Introduction

The initial intuition and motivating question are:

Demands solve tradeoff: more aggressive demands increase proposer's payoff, but also increase prob. of exit, which hurts proposer

After an offer is accepted, proposer may infer that receiver's type is relatively high

Can it be that the proposer is then tempted to make a higher demand, screening out more receiver types? And then continue in this fashion?

Motivation

Three broad areas of application, all related

Crisis bargaining literature in international relations

- Workhorse model: one-shot version of the above $\equiv$ ultimatum game with hidden outside option (Fearon 1995)

Many examples fit the crisis bargaining framework, but are repeated in nature

- Island-building by China in the South China Sea
  - China has also accused the US of "salami slicing" its red line on Taiwan (e.g. through phone calls, Pelosi visit)
- Settlement building in West Bank
- . . .

Motivation

Exploitation in repeated principal-agent relationships

- The proposer (principal) says to the agent in each period: complete this task for me, or you are fired
- If the agent delivers, she may reveal a weak outside option

Muslim rulers setting *jizya* tax on religious minorities (Tirole 2016), or protection rackets

Salami-slicing

If indeed the proposer wants to escalate after screening out some receiver types, his behavior may look like *salami tactics*

> *"Salami tactics," we can be sure, were invented by a child [. . .]. Tell a child not to go in the water and he'll sit on the bank and submerge his bare feet; he is not yet "in" the water. Acquiesce, and he'll stand up; no more of him is in the water than before. Think it over, and he'll start wading, not going any deeper [. . .]. Pretty soon we are calling to him not to swim out of sight, wondering whatever happened to all our discipline.*

Schelling (1965), *Arms and Influence*

Preview of Results

Tirole (Ecta 2016): "From Bottom of the Barrel to Cream of The Crop: Sequential Screening with Positive Selection" studies the *unperturbed model* in which the receiver's outside option is fully persistent

His main result: there is no salami-slicing

- All screening happens in period 1, and demands remain constant thereafter
- Intuition: marginal trade-off faced by the proposer is unaffected by whether inframarginal types have already quit or not—they would quit today either way
- Solution same w/ or w/o commitment

Preview of Results

This paper considers a *perturbed* model in which either the receiver's outside option or the proposer's demands are affected by transient noise

Main insight: with (small) transient noise, the salami may get sliced

- Crucial: shocks not observed up front by proposer
  - Tirole (2016) shows that observed shocks don't do much

Formally, proposals become more aggressive over time, and the probability of exit in the long run is 1

- under some conditions, which depend on whether proposer has commitment power

Preview of Results

Intuition: if I push a little beyond the most aggressive offer that is guaranteed to be accepted, I only get punished if I face a marginal type *and* the shock is really unfavorable

For a small enough push, this effect is second-order

Related Literature

Crisis bargaining: Fearon (1995), Fey Ramsay (2011), Fey Meirowitz Ramsay (2013), Fey Kenkel (2021), Kenkel Schram (2022), ...

- Most of the literature is static
- Fey, Meirowitz and Ramsay (2013): two-shot version with reneging
- Fey and Kenkel (2021): alternating offers+war option

Coase conjecture: Gul, Sonnenschein and Wilson (1986), Myerson (1991)

- Coasian setting: *negative* selection
  - Buyers who stay are low types—invite low prices
  - But low prices break high types' IC constraint if they paid more
  - In our setting, *positive* selection: future offers get worse for the receiver, so no temptation to stick around for them
- Conjecture may fail if:
  - Interdependent values (Deneckere Liang 2006, Fuchs Skrzypacz 2013b), seller cost is Markovian (Ortner 2017), traders or info arrive over time (Fuchs Skrzypacz 2010), seller has private info (Feinberg Skrzypacz 2005), or different terminal payoffs: deadlines (Fuchs Skrzypacz 2013a), outside options (Board Pycia 2014; Hwang Li 2017), or players may "collapse"=lose war (Baliga Sjöstrom 2023)

Related Literature (cont.)

- Other sequential screening:
  - Positive selection + Copts: Tirole (2016), Saleh (2018), Saleh Tirole (2021) (but no unobserved shocks)
  - Coase+Romer Rosenthal: Ali Kleiner Kartik (2023), Evdokimov (2023)
- Endogenous outside option: Fearon 1996, Powell 2006, Schwarz Sonin 2008
  - Leads to a different form of salami-slicing (no private info)
  - Proposer uses front-loaded path of demands
  - Early on, offers are good; later, outside option is bad
- Ratcheting: Laffont Tirole (1988), Hart Tirole (1988)
  - Headline result: agent never reveals type
  - Acharya Ortner (2017): agent may reveal type if environment changes over time
  - Requires big shocks but which may be observed by proposer
  - Logic: revealing type destroys rents only when future environment is good, so may reveal type when future env expected to be bad

The Model

The Model

Time is discrete and finite or infinite: $t = 0, 1, \ldots, T$

for most of this talk, $T = \infty$

Two players, 1 (proposer) and 2 (receiver)

Discount factors $\delta_1, \delta_2 \in [0, 1)$

In each period, 1 makes *demand* $x_t \geq 0$, leading to flow payoffs $(x_t, -x_t - \epsilon_t)$ if accepted by 2, where ϵ_t is a random shock

in the paper, payoffs $(\pi(x_t), -x_t - \beta(x_t)\epsilon_t)$, for π increasing concave, β nondecreasing

The Model

If 2 accepts, flow payoffs accrue and go to next period

If 2 rejects, she takes an outside option and the game ends

- Continuation payoffs $\left(0, -\frac{\theta}{1-\delta_2}\right)$ (in general $-\theta$ per period for player 2)
- θ is persistent and privately observed by 2 at the beginning
- Higher types stay longer: θ measures 2's cost of exit, relative to if 1 makes zero demands

Solution concept: PBE

One-Shot Benchmark

Canonical crisis bargaining model is equivalent to one-shot version ($T = 0$) with no shock ($\epsilon \equiv 0$)

Suppose θ is drawn from a cdf F with support $[\underline{\theta}, \overline{\theta}]$, with $\underline{\theta} > 0$, and F admits a continuous density f

2 accepts a proposal x iff $\theta \geq x$

Then 1's payoff from a demand x is $u(x) := x(1 - F(x))$

One-Shot Benchmark

The optimal x^* satisfies either $0 = u'(x) = 1 - F(x) - xf(x)$ or $x^* = \underline{\theta}$ and $u'(x^*) \leq 0$

If the hazard rate $\frac{f}{1-F}$ is increasing, then there is a unique optimum

Tirole Benchmark

Let's go back to the dynamic model, but still with $\epsilon_t \equiv 0$

Assume parameters s.t. there is a unique optimal demand $x^* = \operatorname{argmax} u$ in the static model

Proposition (Tirole 2016 Props. 1+2+3)

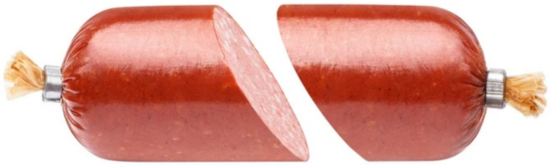
Suppose $T < \infty$. For any δ_1, δ_2, in the unique PBE, the proposer sets $x_t = x^*$ for all t.

If $\delta_1 \geq \delta_2$, then the proposer can do no better with commitment power. (If $\delta_1 < \delta_2$, the proposer can do better under commitment by backloading payoffs.)

Proof

Discussion

All receiver types who quit do so right away



These guys reject right away these guys acquiesce forever

No incentive to slice the salami beyond the first cut

After going to $x_t = x^*$, just as costly as in one-shot setting to push to $x^* + \nu$

The Full Model (With Shocks)

In the main model, shocks ϵ_t are iid with cdf G, satisfying either

A1(η) G admits a density g, symmetric around 0, with support $[-\eta, \eta]$, and s.t. $g|_{[-\eta,\eta]}$ is continuous.

A2(η) G satisfies A1(η) and, in addition, $g(\eta) = 0$.

for some η s.t. $0 < \eta < \underline{\theta}$

Formally equivalent to assume that payoff from rejecting in period t is $-\frac{\theta}{1-\delta_2} + \epsilon_t$

The Full Model (With Shocks)

Two variants of the model:

(i) ϵ_t seen by 2 at beginning of t, and never seen by 1 (unobserved shocks)

(ii) ϵ_t seen by 2 at beginning of t, and by 1 at end of period t (ex post observed shocks)

Some results hold across both cases, will point out when not

Impatient Receiver

Proposition

Suppose $\delta_2 = 0$, $T = \infty$, *and G satisfies A1(*η*) for any* $\eta > 0$. *Then:*

(i) *The proposer's problem is the same with or without commitment.*

(ii) *In any equilibrium (with or without commitment),* $\liminf_{t\to\infty} x_t \geq \overline{\theta} - \eta$ *w.p. 1. Hence, the probability that the receiver exits on the equilibrium path is 1.*

Proof Sketch (for unobserved shocks)

Impatient receiver accepts x_t at t iff $\theta \geq x_t + \epsilon_t$

Then the proposer's problem is

$$\max_x \sum_{t=0}^{\infty} \delta_1^t x_t \int_{\underline{\theta}}^{\overline{\theta}} f(\theta) P_t(\theta; x) d\theta,$$

where $x = (x_t)_t$, and $P_t(\theta; x)$ is the probability that a receiver of type θ accepts all demands *through* period t inclusive:

$$P_t(\theta; x) = \prod_{s=0}^{t} G(\theta - x_s).$$

Part (i) follows from the fact that the choice of $(x_t)_{t \geq s}$ has no impact on the receiver's behavior before period s.

Proof Sketch

If x_t is an interior optimum, it must satisfy the FOC:

$$F_{t+1}(\overline{\theta}; x) = \int_{\underline{\theta}}^{\overline{\theta}} f_t(\theta; x) \left(x_t + \delta_1 U_{t+1}(\theta; x)\right) g(\theta - x_t) d\theta$$

where

$f_t(\theta; x) = f(\theta) P_{t-1}(\theta; x)$ is the density of types at beginning of t

F_t is the associated cdf

$U_t(\theta; x)$ is the proposer's continuation utility at beginning of t cond on demand path and receiver type

As in myopic FOC, LHS is gain from increased demand, RHS is loss from rejections

Proof Sketch

Since $P_t(\theta)$ is weakly decreasing in t for each θ, $P_t(\theta) \searrow P_\infty(\theta)$ for some function $P_\infty(\theta)$

Suppose the exit probability is < 1; equivalently, $P_\infty(\theta) > 0$ for some $\theta < \overline{\theta}$

Let $\theta_\infty = \inf(\text{supp}(P_\infty(\theta)))$. Then $\theta_\infty = \limsup_t x_t + \eta$

For large t, the LHS of the FOC is bounded away from zero

But the RHS goes to zero, since $f_t(\theta; x)$ goes to zero below θ_∞, the proposer's utility is bounded, and $g(\theta - x_t)$ goes to zero for all $\theta > \theta_\infty$

Intuition

For t large enough that most receiver types who *would* have quit have already done so, the proposer can guarantee that virtually no more receiver types *will* quit if he proposes any $x_t \leq \theta_\infty - \eta$

However, if he pushes a little beyond that, demanding $x_t = \theta_\infty - \eta + \nu$ for a small $\nu > 0$, this will only cause exit when the receiver's type is in $[\theta_\infty, \theta_\infty + \nu]$ *and* the shock realization is in $[\eta - \nu, \eta]$, which has probability $\in O(\nu^2)$

The cost of taking this slight risk is thus second-order

The gain, on the other hand, is proportional to ν

Similar argument applies for ex post observed shocks, except that optimal policy is of the form $x_t(h) = x^*(\theta_0)$, where θ_0 is the lowest type left at the beginning of t

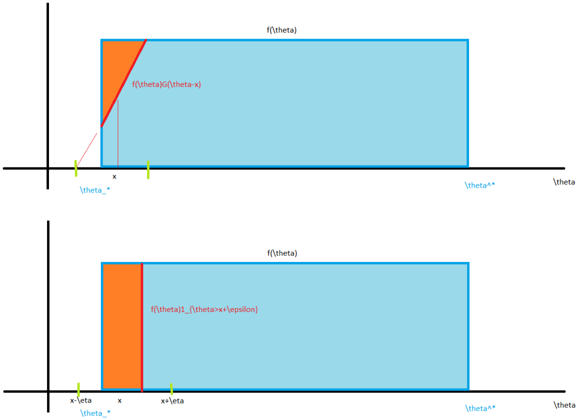


Figure 1: Updating with unobserved or ex post observed shocks

Transition Path

We can give a partial characterization of the transition path

The result is especially simple if F satisfies the MHRP ($\frac{f}{1-F}$ increasing)

In that case, letting x^* be the optimal myopic demand,

Proposition

Take any sequence (η_n) with $\eta_n \searrow 0$, and each G_n satisfying A1(η_n). Take any sequence of demand paths $x^n = (x_t^n)_t$, with x^n optimal for each G_n. Fix $\nu > 0$.

Then, as $n \to \infty$, the proposer never makes demands in $[0, x^* - \nu)$, but spends arbitrarily many periods making demands in any subinterval of $(x^* + \nu, \overline{\theta} - \eta - \nu)$ with positive measure.

Discussion

Under the monotone hazard rate assumption, there is one big initial cut, followed by gradual skimming up to the top of the distribution

These guys exit right away these guys exit eventually

without MHRP...

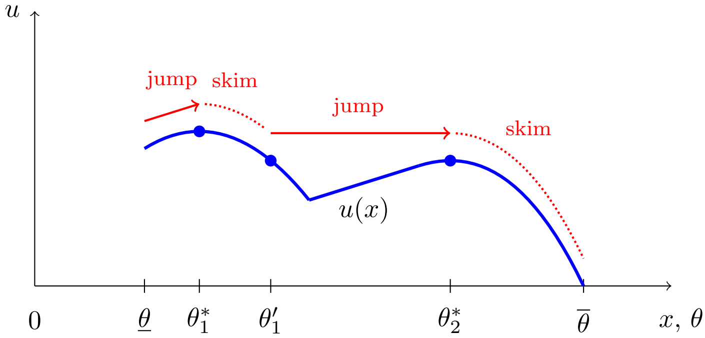


Figure 2: Pattern of escalation with a double-humped f

Major escalations can alternate with slow "skimming" of the distribution

When proposer would not screen anyone in one-shot world, skim

But this can take us back to a world in which we screen out a large set of types $\Longrightarrow$ escalate

Skimming Speed

In the case $\delta_2 = 0$ and if shocks are ex post observed, we can derive an approximate formula for the speed at which the proposer screens out receiver types (when jumps aren't optimal)

After some algebra, we obtain that, if $G \sim U[-\eta, \eta]$, and η is small or δ_1 close to 1, then, if θ is the marginal type, the probability that the receiver quits in the current period (conditional on still being in the game) is approximately

$$\eta \frac{f(\theta)}{1 - F(\theta)} \frac{(1 - \delta_1)^2}{\left(\frac{f(\theta)\theta}{1 - F(\theta)} - \delta_1\right)^2}$$

Skimming speed is

- proportional to η
- related to hazard ratio
- goes to zero as proposer gets patient, or even as periods get shorter

General Discount Factor: Commitment Solution

When $\delta_2 > 0$, the problem differs with vs. without commitment, so consider commitment first

Proposition

Assume G satisfies A1(η) for some $\eta > 0$; $\delta_1 > \delta_2$; and the proposer has commitment power. Let $T = \infty$. Then, under any optimal demand path, the receiver eventually quits with probability 1.

On the other hand, if $\delta_1 \leq \delta_2$, the receiver stays forever with positive probability.

This also holds with observed or unobserved shocks, but proof is easier to state with unobserved shocks

Intuition

Increasing x_t for large t by a small $\nu > 0$ now incurs two different types of costs

- First, some receivers will quit at time t, as in the myopic case—or shortly before
    - but for large t, most receivers who were "on the fence" would have already quit

- So the marginal cost is that some receivers who were previously borderline "sure-accepters" will now be on the fence—this is a mass of the form $K\nu$

- Because they are now only slightly on the fence, they will only actually quit if they also get a bad shock, which happens with prob. $\leq K'\nu$

- So this cost is again of order ν^2

Intuition

Second, some receivers who are on the fence early on might quit at the margin in periods $t' << t$, expecting a lower continuation value far in the future

The assumption $\delta_2 < \delta_1$ ensures that this effect matters little

When $\delta_2 \geq \delta_1$, the first cost still has very little bite, but the second cost becomes big enough to overturn the result

- For large t, fewer "chronic marginal types" are left, but they have been marginal for many periods $\Longrightarrow$ punish over more periods

Proof

General Discount Factor: No Commitment

In the no-commitment case, when choosing x_t, the proposer does not care about the "retroactive" impact of x_t on receiver incentives before t

- As we saw, it is the threat of earlier quitting by "chronic marginal" receivers that can keep the proposer in check
- so, in a no-commitment world, salami slicing should be more likely

But, now the receiver's interpretation of deviations matters (if $\delta_2 > 0$)

- If the proposer deviates off-path from x_t to $x_t + \nu$, what does this imply about future demands?
- They might go up, since a higher x_t screens out more receivers
- Receiver might proactively punish these expected follow-ups

We will focus on the case of ex post observed shocks

Results w/ unobserved shocks

Markovian Equilibria

With ex post obs shocks, proposer knows exactly what types would have accepted a demand, so his posterior is always a truncation of f

Then we can focus on Markovian equilibria, given by $x^*(\theta)$, $\theta^*(y)$

- $x^*(\theta)$ is the equilibrium demand if θ is the lowest type left
- $\theta^*(y)$ is the equilibrium type who is indifferent given an (effective) demand $y = x + \epsilon$
- Then $\theta^*_{t+1} = \max(\theta^*_t, \theta^*(x^*(\theta^*_t) + \epsilon_t))$

We refer to eqs with these properties simply as *equilibria*

Note: in such equilibria, it does not matter if receiver sees x_t, ϵ_t or only $x_t + \epsilon_t$

Equilibrium with No Commitment

Proposition

Assume that G satisfies A2(η) and $T = \infty$. For any $\delta_1, \delta_2 \in [0, 1)$, in any equilibrium, the receiver eventually quits with probability 1.

Moreover, if G only satisfies A1(η), the same result holds in any continuous equilibrium, that is, any equilibrium in which x^* and θ^* are continuous.

Intuition

Suppose $\lim \theta_t^* < \overline{\theta}$ with positive probability

Take t large enough that we are "close to the limit"

What's the cost of a small deviation ν?

Under A2(η), probability $o(\nu)$ that it even leads to an effective demand that *could* cause exit

Under A1(η), this probability is $O(\nu)$, but the probability that a receiver actually quits conditional on such an effective demand is low for small ν, due to the continuity of θ^*

A Closed Form Example

Suppose F follows a power law: $F(\theta) = 1 - \left(\frac{\theta_0}{\theta}\right)^{\alpha}$ for some $\theta_0 > 0$, $\alpha > 1$

Note $1 - F = \left(\frac{\theta_0}{\theta}\right)^{\alpha}$, $f(\theta) \propto \frac{1}{\theta^{\alpha+1}}$

Suppose in addition that flow payoffs if a demand x_t is accepted and a shock ϵ_t is realized are $(x_t, -x_t - x_t\epsilon_t)$, i.e., $\beta(x) \equiv x$ (multiplicative shocks)

A Closed Form Example

The environment is stationary (up to normalization) as the marginal type increases, so there is a *stationary* equilibrium:

$x^*(\theta) \equiv x_0\theta$, $\theta^*(y) \equiv y(1-\omega)$ for some x_0, ω

Let $z_0 = x_0(1-\omega)$, and ϵ^* be the marginal ϵ for which no receiver types quit on path, i.e., $x_0\theta(1+\epsilon^*)(1-\omega) = \theta$, so $\epsilon^* = \frac{1}{z_0} - 1$

A Closed Form Example

Then we can show that the receiver's Bellman equation boils down to:

$$\frac{\omega}{\delta_2} = -z_0 \int_{-\eta}^{\epsilon^*} g(\epsilon)(1+\epsilon)d\epsilon + G(\epsilon^*)$$

Moreover, after much algebra, the proposer's equilibrium condition pinning down x_0 implies:

$$\frac{\frac{1-\eta}{(1+\epsilon^*)^2} - \frac{(1+\epsilon^*)^{\alpha-2}}{(1+\eta)^{\alpha-1}}}{\frac{\eta+\epsilon^*}{1+\epsilon^*} + \frac{1}{\alpha-1}\left(1-\left(\frac{1+\epsilon^*}{1+\eta}\right)^{\alpha-1}\right)} = \delta_1 \frac{\alpha-1}{\alpha-2} \frac{\left(\frac{1+\epsilon^*}{1+\eta}\right)^{\alpha-2} - 1}{2\eta - \delta_1(\eta+\epsilon^*) - \frac{\delta_1}{\alpha-2}\left(1+\epsilon^* - \frac{(1+\epsilon^*)^{\alpha-1}}{(1+\eta)^{\alpha-2}}\right)}$$

Crucially, z_0 (hence ϵ^*) is pinned down by an equation where δ_2 does not show up

Hence ϵ^*, z_0 indep of δ_2, so $\omega = \omega_0\delta_2$ for some ω_0 indep of δ_2

A Closed Form Example

Takeaway: as the receiver gets more patient, she is less prone to quitting due to putting weight on option value of staying

Then $x_0 = \frac{z_0}{1-\delta_2\omega_0}$ is increasing in δ_2: the proposer takes advantage!

Effects cancel out, so evolution of the state is independent of δ_2

Pushing the No-Commitment Result

Can we strengthen the result in the Proposition to show unraveling with just A1(η)?

Requires showing existence of a continuous equilibrium, or putting a bound on size of discontinuities

Claim:

$$y - \delta_2 \eta \leq \theta^*(y) \leq y$$

If $\theta > y$, myopically optimal to stay; can always quit later

Can show that type $\theta = y - \delta_2 \eta$ is indifferent about quitting today if facing demand y and $x = (y - \eta) + \epsilon$ in all future periods... but path of x_t's never decreases and today's x must've been at least $y - \eta \implies$ future x's at least $y - \eta$

Discontinuities in $\theta^*(\cdot)$ are of size at most $\delta_2 \eta$!

Pushing

Suppose that $\theta^*(x^*(\theta) + \eta) \leq \theta$ for some θ: never skim past θ

Suppose proposer deviates to $x^*(\theta) + \nu$ for $\nu > 0$ small

w.p. $\approx 1 - g(\eta)\nu$, $\epsilon \leq \eta - \nu \implies y \leq x^*(\theta) + \eta$: nothing happens and proposer gains ν extra

"w.p." $g(\epsilon)$ for each $\epsilon \in [\eta - \nu, \eta]$, $y = x^*(\theta) + \nu + \epsilon$

> w.p. $\approx \frac{f(\theta)}{1-F(\theta)}(\theta^*(x^*(\theta) + \nu + \epsilon) - \theta)$, receiver quits; proposer loses $U(\theta) \leq M$
>
> w.p. $\approx 1 - \cdot$, receiver stays; state moves up; proposer gains a nonzero amount

Still pushing

Because of "small discontinuities" and the assumption $\theta^*(x^*(\theta)+\eta) \le \theta$,

$$\begin{aligned}\theta^*(x^*(\theta)+\nu+\epsilon)-\theta &\le \theta^*(x^*(\theta)+\nu+\epsilon)-\theta^*(x^*(\theta)+\eta)\\ &\le x^*(\theta)+\nu+\epsilon-(x^*(\theta)+\eta-\delta_2\eta) =\\ &= \nu+\epsilon-(1-\delta_2)\eta\end{aligned}$$

The integral of this over all $\epsilon \in [\eta-\nu,\eta]$ equals $\delta_2\nu\eta+\frac{\nu^2}{2}$

So the net gain from the deviation is at least (approximately)

$$\begin{aligned}&\nu-\frac{f(\theta)}{1-F(\theta)}g(\eta)(\delta_2\nu\eta+\frac{\nu^2}{2})M-g(\eta)\nu^2\\ =&\nu-\frac{f(\theta)}{1-F(\theta)}Mg(\eta)\delta_2\nu\eta+O(\nu^2)\end{aligned}$$

Still pushing

Hence this deviation is profitable for small $\nu > 0$ if

$$1 > \delta_2 M \frac{f(\theta)}{1 - F(\theta)} g(\eta)\eta$$

Conclusion: holding everything else fixed (including η), if we change g to make $g(\eta)$ lower, eventually there is skimming through at least some part of the distribution (until $1 - F(\theta)$ gets small)

If we "shrink" noise (multiply shocks by $\alpha < 1$, so $\tilde{g}(\epsilon) = \frac{g(\alpha\epsilon)}{\alpha}$) then $g(\eta)\eta$ does not change $\Longrightarrow$ still get the same result

Compare with "known type" case ($\theta \equiv \theta_0$): if there is idiosyncratic noise ϵ and $g(\eta)$ is very low, may still choose to risk quitting, but incentive to do so vanishes for α small enough

Extensions

Give receiver more nuanced actions rather than simply accept vs exit: ✓
details

In practice, the target may make the provocateur back off with a show of force (short of war)

We show: main results survive if receiver can take intermediate actions that allow a positive risk of war (under some conditions)

Consider more general contracts in commitment case: in progress details

Preliminary results indicate that, with general contracts, receiver still induced to quit w.p. 1 in the long run under A2, but not necessarily under A1

More results on no-commitment case: in progress

Discussion

Noise in repeated screening can have drastic effects

However, these effects depend on certain conditions, besides A1(η)

- With commitment: need $\delta_2 < \delta_1$
- Without commitment: *probably* hinges on some combination of A2(η) and/or "smoothness" of equilibrium behavior

Discussion

If we think exit is a bad outcome, what can prevent it and/or ameliorate salami-slicing?

- As in one-shot case, lower variance in F, or more right-skewed distribution, helps
- Lower noise helps: better if receiver's preferences are stable over time
- Better if noise is observable by the sender (cf. Tirole 2016): transparency is good
  - E.g., can proposer observe and understand the receiver's domestic political circumstance, mapping into audience costs?
- Better if proposer's intent is understood by receiver
  - Makes drawing red lines easier (Schelling 1965, Dong 2023)
- Better if demands are frequent, at least when receiver is impatient
  - One excessive demand causes exit, but it doesn't last long

Thank you!

Proof Sketch

Assume no commitment power, and $T < \infty$

Note: any equilibrium demand path must be non-decreasing

Proof w/ 2 periods: suppose $x_0 > x_1$

In $t = 0$, receivers with $\theta < \frac{x_0 + \delta_2 x_1}{1+\delta_2}$ screen out

Then, in $t = 1$, no reason to demand less than $\frac{x_0 + \delta_2 x_1}{1+\delta_2} > x_1$, a contradiction

For any non-decreasing demand path, receivers respond myopically: quit at t iff $\theta < x_t$

Then a path $(x_0, \ldots, x_T)$ obtains a payoff $\sum_{t=0}^{T} \delta_1^t u(x_t) \leq \sum_{t=0}^{T} \delta_1^t u(x^*)$

Show by backward induction that proposer indeed proposes x^* in every period in PBE

Proof Sketch

With commitment power, demanding x^* in every period is still the best non-decreasing demand path

The previous argument no longer rules out non-monotonic paths, but this does if $\delta_2 \leq \delta_1$
Suppose $x_t > x_{t+1}$. What changes if 1 instead proposes $\tilde{x} = \tilde{x}_{t+1} = \frac{x_t + \delta_2 x_{t+1}}{1+\delta_2}$?

2 weakly more willing to accept at t (good for 1) and hence earlier

2 less willing to accept at $t+1$, but it never matters!

> If 2 accepts at t, either $\theta \geq \tilde{x}_t$ (done), or $\theta < \tilde{x}_t$ but $\theta \geq \frac{\sum_{s=t}^{l} \delta^{s-t} x_s}{\sum_{s=t}^{l} \delta^{s-t}}$ for some $l \geq t+2 \Longrightarrow \theta \geq \frac{\sum_{s=t+1}^{l} \delta^{s-t} \tilde{x}_s}{\sum_{s=t+1}^{l} \delta^{s-t}}$

With such "flips" we can eventually make the proposal path weakly increasing

These flips are a strict improvement for the proposer if $\delta_2 < \delta_1$ or at worst indifferent if $=$

Return

Proof Sketch

Take $T = \infty$

Let $V_t(\theta; x)$ be the continuation payoff of a receiver of type θ at the beginning of period t:

$$V_t(\theta; x) = E_{\epsilon_t}\left[V_t(\theta; x, \epsilon)\right], \text{ where}$$

$$V_t(\theta; x, \epsilon) = \max\left[-x_t - \epsilon_t + \delta_2 V_{t+1}(\theta; x), -\frac{1}{1-\delta_2}\theta\right]$$

Now the receiver's decision rule takes continuation payoffs into account:

$$P_{t,t}(\theta; x) = G\left(\frac{1}{1-\delta_2}\theta - x_t + \delta_2 V_{t+1}(\theta; x)\right)$$

$$\frac{\partial P_{t,t}(\theta; x)}{\partial x_s} = \begin{cases} -g\left(\frac{1}{1-\delta_2}\theta - x_t + \delta_2 V_{t+1}(\theta; x)\right) \text{ if } s = t \\ -g\left(\frac{1}{1-\delta_2}\theta - x_t + \delta_2 V_{t+1}(\theta; x)\right) \delta_2^{s-t} P_{s,t+1}(\theta) \text{ if } s > t \end{cases}$$

where $P_{s,t}(\theta; x)$ is receiver's prob of accepting through s conditional on being in the game at t

Proof Sketch

Now the proposer's FOC at time t is:

$$F_{s+1}(\overline{\theta}; x) = \sum_{t=0}^{s} \left(\frac{\delta_2}{\delta_1}\right)^{s-t}$$

$$\int_{\underline{\theta}}^{\overline{\theta}} f(\theta) P_{t-1}(\theta; x) g\left(\frac{\theta}{1-\delta_2} - x_t + \delta_2 V_{t+1}(\theta; x)\right) P_{s,t+1}(\theta) \overline{U}_t(\theta; x) d\theta$$

where $\overline{U}_t(\theta; x) = x_t + \delta_1 U_{t+1}(\theta; x)$

Terms for $t < s$ on RHS measure "proactive" punishments by receiver

Proof Sketch

For $\delta_2 < \delta_1$, we can separately bound, choosing k appropriately:

$t = s - k, s - k + 1, \ldots, s$ for fixed k: handle it similarly to $t = s$ as in myopic case (bounded number of terms)

$t < s - k$: very small because $\left(\frac{\delta_2}{\delta_1}\right)^{s-t}$ goes to 0 exponentially

Then RHS $\xrightarrow[s\to\infty]{} 0$, so LHS also goes to zero $\Longrightarrow$ receiver eventually quits

Proof Sketch

For $\delta_2 \geq \delta_1$, all terms on RHS matter for large s

E.g., for G uniform, we can bound the RHS below by an expression of the form:

$$\underline{f\,U g} \int_{\underline{\theta}}^{\overline{\theta}} P_s(\theta; x) \#\{t \leq s : \theta \text{ is marginal at } t\} d\theta$$

We can show that this either goes to zero slower than the LHS, or it is bounded away from zero

> e.g. for flavor: if $x_t \equiv \overline{\theta} - (1-\delta_2)\eta$, so all types in $[\overline{\theta} - (2-\delta_2)\eta, \overline{\theta}]$ are marginal forever, the integral becomes similar to $\int_{\underline{\theta}}^{\overline{\theta}} \left(1 - \frac{\theta}{\overline{\theta}}\right)^{s+1} (s+1) d\theta \sim \frac{s+1}{s+2} \nrightarrow 0$

Hence, LHS cannot go to zero

Return

Salami Slicing in Smooth Equilibria

Say an equilibrium (PBE) is *smooth* if, for all t, at any history $h^t = (x_0, \ldots, x_t)$, the continuation demand path $(x_{t+1}, x_{t+2}, \ldots)(h^t)$ is such that x_s is a differentiable function of x_t for all s, and these derivatives are uniformly bounded for all s, t.

Proposition

Assume G satisfies A1(η) and $T = \infty$. Then, for any δ_1, $\delta_2 < 1$, the receiver quits with probability 1 in any smooth equilibrium.

Intuition

Suppose for a moment that the receiver's expectation of x_s $(s > t)$ were unaffected by a change in x_t

Then the proposer's FOC for x_s would be:

$$F_{s+1}(\overline{\theta}; x) = \sum_{t=0}^{s-1} \left(\frac{\delta_2}{\delta_1}\right)^{s-t}$$

$$\int_{\underline{\theta}}^{\overline{\theta}} f(\theta) P_{t-1}(\theta; x) g\left(\frac{\theta}{1-\delta_2} - x_t + \delta_2 V_{t+1}(\theta; x)\right) P_{s,t+1}(\theta) \overline{U}_t(\theta; x) d\theta$$

$$+ \int_{\underline{\theta}}^{\overline{\theta}} f(\theta) P_{s-1}(\theta; x) g\left(\frac{\theta}{1-\delta_2} - x_s + \delta_2 V_{s+1}(\theta; x)\right) \overline{U}_s(\theta; x) d\theta$$

Intuition

Then a similar proof would work as in the commitment case with $\delta_2 < \delta_1$, but now regardless of δ_2

When the receiver's expectations adjust to deviations, the RHS is multiplied by a factor bounded by $\sum_{l \geq s} \delta_2^{l-s} \frac{\partial x_l}{\partial x_s}$

Hence bounded if the derivatives are uniformly bounded

However, whether all equilibria are smooth, or a smooth equilibrium even exists, are open questions at this point

Opaque Deviations

With no commitment and $\delta_2 > 0$ it matters whether the receiver sees both x_t and ϵ_t at time t, or only $x_t + \epsilon_t$, the *effective demand*

If only $x_t + \epsilon_t$ is observed, the receiver would often be unaware of a small deviation

- If proposer deviates $x_t \to x_t + \nu$, but $\epsilon_t \in [-\eta, \eta - \nu]$, receiver has no clue

- Even if $\epsilon_t > \eta - \nu$, interpretation is not obvious: both a deviation and $\epsilon_t \notin [-\eta, \eta]$ are prob. 0 events

- We say that the receiver is *naive* if, even when the effective demand is outside of $[x_t - \eta, x_t + \eta]$, she believes the proposer has not deviated (and hence will not deviate further) but that a rare shock materialized

Note: lack of awareness or naivete do **not** imply no response to deviation, only no response to *further* deviations that might be expected

Opaque Deviations and Naivete

Proposition

Assume that $T = \infty$ and that the receiver observes only effective demands in each period. Then, if either

(i) *G satisfies A1(η) and the receiver is naive, or*

(ii) *G satisfies A2(η),*

then, for any $\delta_1, \delta_2 \in [0, 1)$, in any PBE, the receiver eventually quits with probability 1.

Intuition

When the receiver is naive, the proposer FOC written in our previous intuition is the relevant one (it is as if $\frac{\partial x_l}{\partial x_s} = 0$ given the receiver's behavior)

When the receiver is not naive, but only observes effective demands and A2(η) is satisfied, the probability that a deviation of size ν is even detected is of size $o(\nu)$ for $\nu \to 0$

- Then, again, small deviations "after marginal types have left" are virtually unpunishable
- The proposer is not dissuaded even if *any* detection of a deviation leads to sure exit

Return

Richer Receiver Actions

Baseline model is quite stark: receiver has to either end the game or let the proposer do whatever

Anecdotally, salami slicing is sometimes countered by a show of force that falls short of ending the game

- "show you mean business"

What happens if we allow intermediate responses by receiver?

Intuitively, two things

- receiver can now act tough $\Longrightarrow$ signaling concerns may induce *more* aggressive receiver behavior
- in equilibrium, more info about receiver's type may be revealed $\Longrightarrow$ proposer may be able to better tailor demands, leading to less exit

Setup

We operationalize “intermediate" actions in a simple way

Now receiver has access to a (finite) set of quitting probabilities $0 = p^0 < \ldots < p^k = 1$ $(k \geq 2)$

Choosing p_i means the game ends w.p. p_i; w.p. $1 - p_i$, game continues and acceptance flow payoffs accrue

Crucially, proposer sees p_i (otherwise it's just mixing): rolling the dice signals toughness even if you end up rolling peace

A Negative Result

Take $T = \infty$ throughout.

Proposition

Assume commitment power; A1(η) for some $\eta > 0$; and $\delta_1 > \delta_2$. Then, under any optimal demand path, the receiver eventually quits w.p. 1.

Proposition

Assume no commitment power; A2(η) for some $\eta > 0$; and the receiver observes only effective demands in each period. Then, for any $\delta_1, \delta_2 \in [0, 1)$, in any PBE, the receiver eventually quits w.p. 1.

Proposition

Assume no commitment power and A2(η) for some $\eta > 0$ in the "ex post observed shocks" setting. Then, for any $\delta_1, \delta_2 \in [0, 1)$, in any equilibrium, the receiver eventually quits w.p. 1.

Note: now the receiver may quit w.p. 1 either by quitting outright ($p = 1$) once or by rolling the dice ($p > 0$) infinitely many times!

Proof

Return

Proof Sketch: Commitment

A path of play for the receiver is a sequence $S = (p_t)_t$, either infinite with $p_t \in \{p^0, \ldots, p^{k-1}\}$ for all t...

type 1 if finitely many nonzero elements

type 2 otherwise

or finite, with (only) the last term equal to 1 (type 3)

Let $P(\theta, S)$ be the probability that a receiver of type θ plays according to S on path (assuming that all dice rolls lead to peace)

Let $P(S) = \int P(\theta, S) dF(\theta)$

Then we need to show that $P(S) = 0$ for all type 1 sequences S

Proof Sketch: Commitment (Cont.)

Suppose otherwise, and let S_0 be a sequence with minimal number of nonzero elements among those with $P(S) > 0$

Denote by $S|t$ a sequence truncated to size t

Note that a proposer strategy can be described by a function $x_t(S^t)$ defining a demand x for all length-t sequences S^t with no 1's

Taking $t > \max\{s : S_0(s) > 0\}$, consider a deviation by the proposer from the equilibrium x to $\tilde{x}$ with $\tilde{x}_t(S_0|t) = x_t(S_0|t) + \nu$ and $\tilde{x} \equiv x$ elsewhere

- i.e., demand ν more at time t if receiver has been playing according to S_0

Proof Sketch: Commitment (Cont.)

We will argue that this deviation is profitable if ν small and t large are appropriately chosen, a contradiction

The gain: (at least) $P(S_0)\nu$

The loss: (at most) M (max proposer loss from exit) $\times$ $(\sum_{s \le t} \delta_1^{s-t} Q_s)$, where Q_s is the additional *switching* prob at time t

- conditional on reaching period $t+1$ with unchanged actions, receiver will then behave the same
- but now, besides quitting at $s \le t$, receiver may also switch (e.g. roll the dice vs not, or roll harder)
- we bound all proposer payoff changes from such switches by the max loss from receiver quitting

Proof Sketch: Commitment (Cont.)

Need to bound Q_s—strategy is similar to baseline model

Write $Q_s = \sum_{S^s} Q_s(S^s)$, where Q_s is switching prob when receiver has played according to S^s so far

If $S^s \neq S_0|s$, then receiver unaffected by deviation $\Longrightarrow Q_s(S^s) = 0$

So just need to look at $Q_s(S_0|s)$ for $s \leq t$

For $s = t - m, \ldots, t$, use the fact that, if t large enough, almost no receivers would quit in absence of a deviation

- because remaining receivers are almost sure to stay on path, eq path payoff is almost flat in θ, whereas continuation payoff from rolling dice (or quitting) has strictly positive slope
- lowest receivers left in support are at least marginally willing to choose $p = 0$
- then all higher receivers are sure stayers—even if a little more is taken

$\Longrightarrow$ for fixed m, this converges to $O(\nu^2)$ as $t \to \infty$

Proof Sketch: Commitment (Cont.)

For $s < t - m$, use that receiver prefers p^i to p^j iff a condition of the form $V_i - V_j + \epsilon_t(p^j - p^i) > 0$ holds

If p_i corresponds to sticking to S_0, then V_i is a function of ν (with bounded derivative, with a bound of the form $D\delta_2^{t-s}$); if not, independent

Then $Q_s(S_0|s) \leq k \frac{D\delta_2^{t-s}}{\min|p^{i+1} - p^i|} \overline{f}\nu$

Picking m large enough, we can make these terms smaller than the gain, using that $\delta_2 < \delta_1$

Proof Sketch: No Commitment

In the no commitment cases, the result is even simpler: as in the original model with binary receiver action, an increase of ν in the current demand goes unpunished w.p. going to $1 - o(\nu)$ as $t \to \infty$

What happens if receiver's action is continuous (can choose any $p \in [0, 1]$?) No idea

Return

More General Contracts

In the commitment case, what happens if the proposer has access to general contracts?

- Rather than committing to a demand path $(x_t)_{t \geq 0}$, commit to a dynamic menu $(X_t(\cdot))_{t \geq 0}$, where $X_0 \subseteq \mathbb{R}_{\geq 0}$ and in general $X_t(x_0, \ldots, x_{t-1}) \subseteq \mathbb{R}_{\geq 0}$
  - Receiver may get to choose payoff today, but with future consequences

Proposition

Assume G satisfies A2(η) for some $\eta > 0$; $T = \infty$; and $\delta_1 > \delta_2$. Suppose the proposer has commitment power and access to general contracts. Then the receiver eventually quits w.p. 1.

Proof

Return

Proof Sketch

First, an observation: it is without loss to focus on direct revelation mechanisms where the receiver reveals θ and then the proposer applies a demand path $(x_t(\theta))_{t \geq 0}$ satisfying IC constraints. Why?

> Define $x_t(\theta)$ to be the path θ would choose in equilibrium, assuming she doesn't quit
>
> Crucially, even though receiver gets interim info about payoffs (ϵ_t only realized at time t), these *do not affect* ranking of non-exit options
>
> So, in absence of mixing, θ's preferred path of transfers is predictable

Let $P_t(\theta)$ ($Q_t(\theta)$) be θ's prob of accepting through (in) period t, now in response to her personalized demand path

Let $V(\tilde{\theta}; \theta)$ be the receiver's value function ex ante from reporting $\tilde{\theta}$ if her true type is θ

Proof Sketch

Type θ's IC constraint implies $\frac{\partial}{\partial 1} V(\theta;\theta) = 0$, and it is more or less enough to check just this for all θ

The envelope theorem follows: $\frac{d}{d\theta} V(\theta;\theta) \equiv \frac{\partial}{\partial 2} V(\theta;\theta)$

$\frac{\partial}{\partial 2} V(\theta;\theta) = -\sum_{t\geq 0} \delta_2^t (1 - P_t(\theta))$

Suppose that the proposer wants to implement an equilibrium generating a given value function $V(\theta)$ ($\equiv V(\theta;\theta)$). Can he? How?

Yes

Need to choose, for each θ, a path $(x_t(\theta))_t$ with two properties: best response by receiver indeed nets her the payoff $V(\theta;\theta)$, and need to get the derivative right, i.e., $\sum_{t\geq 0} \delta_2^t P_t(\theta)$ is pinned down

Proof Sketch

Subject to these constraints, there are still many degrees of freedom in choosing $(x_t(\theta))_t$

The principal then needs to solve, for each θ, an optimization problem of the form

$$\max_{(x_t(\theta))_t} \sum_{t=0}^{\infty} \delta_1^t \pi(x_t(\theta)) P_t(\theta)$$

subject to: $V(\theta;\theta) = \overline{V}(\theta)$ and $\sum_{t\geq 0} \delta_2^t P_t(\theta) = \overline{V}'(\theta)$ for a given function $\overline{V}$

Set up the Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \delta_1^t \pi(x_t(\theta)) P_t(\theta) + \lambda(\overline{V} - V(\cdot)) + \mu(\overline{V}' - \sum_{t=0}^{\infty} \delta_2^t P_t(\theta))$$

where $V(\cdot)$ is θ's equilibrium utility given the demand path $x(\theta)$

Proof Sketch

Consider a deviation of the form: x_{t-1} changes by $-\nu\delta_2 Q_t$, x_t changes by ν, for $\nu > 0$ small

What is the impact on the receiver's value function V_s in each period?

For $s > t$: $\frac{\partial V_s}{\partial \nu} = 0$ trivially

For $s = t$: $\frac{\partial V_t}{\partial \nu} = -Q_t$ by envelope theorem

For $s = t-1$: $\frac{\partial -x_{t-1}+\delta_2 V_t}{\partial \nu} = \delta_2 Q_t - \delta_2 Q_t = 0$

Then $\frac{\partial V_{t-1}}{\partial \nu} = 0$, so $\frac{\partial V_s}{\partial \nu} = 0$ for all $s < t$

Proof Sketch

What is the impact on Q_s in each period?

For $s > t$: $\frac{\partial Q_s}{\partial \nu} = 0$ trivially

For $s = t$: $\frac{\partial Q_t}{\partial \nu} = -g\left(\frac{\theta}{1-\delta_2} - x_t + \delta_2 V_{t+1}\right) =: -g_t$

For $s < t$: $\frac{\partial Q_s}{\partial \nu} = 0$ because there is no change to $-x_{t-1} + \delta_2 V_t$ and no change for x_{s-1}, V_s for $s < t$

Then what is the impact on P_s?

For $s < t$: $\frac{\partial P_s}{\partial \nu} = 0$

For $s = t$: $\frac{\partial P_t}{\partial \nu} = P_{t-1}\frac{\partial Q_t}{\partial \nu} = -P_{t-1}g_t = -\frac{g_t}{Q_t}P_t$

For $s > t$: $\frac{\partial P_s}{\partial \nu} = P_s \frac{\frac{\partial P_t}{\partial \nu}}{P_t} = -\frac{g_t}{Q_t}P_s$

Proof Sketch

Then

$$\frac{\partial \mathcal{L}}{\partial \nu} = -\delta_1^{t-1}\pi'_{t-1}\delta_2 Q_t P_{t-1} + \delta_1^t \pi'_t P_t + \sum_{s\geq t}\delta_1^s \pi_s \frac{\partial P_s}{\partial \nu} - \mu \sum_{s\geq t}\delta_2^s \frac{\partial P_s}{\partial \nu}$$

$$\frac{1}{\delta_1^t}\frac{\partial \mathcal{L}}{\partial \nu} = (-\frac{\delta_2}{\delta_1}\pi'_{t-1} + \pi'_t)P_t + \frac{g_t}{Q_t}\left(-\sum_{s\geq t}\delta_1^{s-t}\pi_s P_s + \mu \sum_{s\geq t}\left(\frac{\delta_2}{\delta_1}\right)^t \delta_2^{s-t} P_s\right)$$

If the allocation is optimal, the RHS should vanish for all t

Suppose type θ stays forever with positive probability, so $P_s \searrow P_\infty > 0$

Then $Q_t \to 1$

$\mu \sum_{s\geq t}\left(\frac{\delta_2}{\delta_1}\right)^t \delta_2^{s-t} P_s$ is bounded above by $\mu \left(\frac{\delta_2}{\delta_1}\right)^t \frac{1}{1-\delta_2} P_\infty \xrightarrow[t\to\infty]{} 0$

Proof Sketch

$$\frac{1}{\delta_1^t}\frac{\partial \mathcal{L}}{\partial \nu} = (-\frac{\delta_2}{\delta_1}\pi'_{t-1} + \pi'_t)P_t + \frac{g_t}{Q_t}\left(-\sum_{s\geq t}\delta_1^{s-t}\pi_s P_s + \mu\sum_{s\geq t}\left(\frac{\delta_2}{\delta_1}\right)^t \delta_2^{s-t}P_s\right)$$

$\sum_{s\geq t}\delta_1^{s-t}\pi_s P_s$ is bounded above by $\frac{\overline{\theta}+\eta}{1-\delta_2}\frac{1}{1-\delta_1}P_\infty$, so the limsup of the second term in absolute value is no more than $\frac{\overline{\theta}+\eta}{1-\delta_2}\frac{1}{1-\delta_1}P_\infty \limsup_t g_t$

But, if $Q_t \to 1$, then $g_t \to 0$ by A2(η), so this also goes to zero

Then $-\frac{\delta_2}{\delta_1}\pi'_{t-1} + \pi'_t \to 0$, which implies that $\pi'_t \to 0$—this is either impossible or leads to $x_t \to \infty$, contradicting $P_\infty > 0$

Return