Corrigendum to "Deliver the Vote! Micromotives and Macrobehavior in Electoral Fraud"

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This corrigendum identifies errors in the calculation of the equilibrium reward w^* in the original article (Rundlett and Svolik, 2016). Its authors, Ashlea Rundlett and Milan Svolik, regret their inclusion and would like to thank German Gieczewski and Mehdi Shadmehr for identifying them.

- The correct w^* for the baseline model (p. 186) is

$$w^* = \begin{cases} \sqrt{\frac{2cF(b+2\epsilon c+cF)}{1+F(1+2\epsilon+2F)}} - c & \text{if } b > c\left(\frac{1+F(1-2\epsilon)}{2F}\right); \\ 0 & \text{otherwise.} \end{cases}$$

- The correct w^* for the "fraud as insurance" extension (p. 188) is

$$w^* = \begin{cases} \sqrt{\frac{bFc+F^2c^2+2\epsilon Fc^2}{2\sigma\hat{\theta}+\left[\hat{\theta}+\sigma-\frac{1}{2}\right]F+F^2+F\epsilon}} - c & \text{if } b > c\left(2\frac{\sigma\hat{\theta}}{F}+\hat{\theta}+\sigma-\frac{1}{2}-\epsilon\right); \\ 0 & \text{otherwise.} \end{cases}$$

- In the "differences in competitiveness" extension (p. 189), the payoff offered by the incumbent should be $w[R_i - \alpha P_i]$, and the correct w^* (p. 8 in the supplementary

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appendix) is

$$w^* = \begin{cases} \sqrt{\dfrac{\frac{bFc}{1-\alpha}+\frac{F^2c^2}{1-\alpha}+2c^2\epsilon F}{\frac{1-\alpha}{2}+\left(1-\frac{\frac{1}{2}-\alpha\pi-F}{1-\alpha}+\epsilon\right)F}} - c & \text{if } b > c\left[\frac{(1-\alpha)^2}{2F}+\frac{1}{2}-\alpha(1-\pi)-\epsilon(1-\alpha)\right]; \\ 0 & \text{otherwise.} \end{cases}$$

Implications Resolving the error in the calculation of the equilibrium reward factor w^* does not change key, substantive insights of the original model and is not relevant for the paper's empirical analysis. The two key equilibrium thresholds, θ^* and S^*, remain correct and are not affected. Correcting this error, however, is consequential for the indirect effect on the equilibrium level of fraud of two parameters, ϵ and F. The derivative of θ^* with respect to ϵ is positive for large enough values of b after correction, implying that when the incumbent's payoff exceeds a threshold, a greater district "heterogeneity" *reduces* equilibrium levels of fraud. Meanwhile, after correction, the derivative of θ^* with respect to F is too complicated to yield politically useful insights about the indirect effect of F on θ^*.

Detailed Analysis

Baseline Model The solution to the incumbent's optimal choice of $w \geq 0$ in the baseline model presented in section A.3 of the supplementary appendix was based on the optimization problem

$$\max_w b(1-\theta^*) - w\left(\frac{1}{2} + F\int_0^1 \frac{(\theta+\epsilon)-S^*}{2\epsilon}\,d\theta\right).$$

This formulation did not account for the fact that, as outlined on p. 186, no agent engages in fraud in equilibrium when $\theta < S^* - \epsilon$, while all agents engage in fraud in equilibrium when $\theta > S^* + \epsilon$. In sum, the fraction of agents ϕ that engage in fraud in equilibrium is

$$\phi = \begin{cases} 0 & \text{if } \theta < S^* - \epsilon; \\ \frac{(\theta+\epsilon)-S^*}{2\epsilon} & \text{if } S^* - \epsilon \leq \theta \leq S^* + \epsilon; \\ 1 & \text{if } \theta > S^* + \epsilon. \end{cases}$$

Accounting for the above and taking an expectation with respect to the distribution of θ, the incumbent solves

$$\max_w b(1-\theta^*) - w\left(\frac{1}{2} + F\left[\int_0^{S^*-\epsilon} 0\, d\theta + \int_{S^*-\epsilon}^{S^*+\epsilon} \frac{(\theta+\epsilon)-S^*}{2\epsilon}\, d\theta + \int_{S^*+\epsilon}^{1} 1\, d\theta\right]\right).$$

Treating θ^* and S^* as functions of w, the above simplifies to

$$\max_w b\left(\frac{1}{2} + F\frac{w}{c+w}\right) - w\left(\frac{1}{2} + F\left[\epsilon + \frac{1}{2} + \frac{Fw - 2\epsilon c}{c+w}\right]\right),$$

where we used

$$\int_0^{S^*-\epsilon} 0\, d\theta = 0, \quad \int_{S^*-\epsilon}^{S^*+\epsilon} \frac{(\theta+\epsilon)-S^*}{2\epsilon}\, d\theta = \epsilon, \quad \text{and} \int_{S^*+\epsilon}^{1} 1\, d\theta = \frac{1}{2} + \frac{Fw - 2\epsilon c}{c+w},$$

as well as the solutions for θ^* and S^* stated on p. 185,

$$\theta^* = \frac{1}{2} - F\frac{w}{c+w} \quad \text{and } S^* = \frac{1}{2} - F\frac{w}{c+w} + \epsilon\frac{c-w}{c+w}. \tag{1}$$

The first-order condition for this optimization problem is

$$b\left(-\frac{1-2F}{2(c+w)} + \frac{c+w-2Fw}{2(c+w)^2}\right) -$$
$$\left(\frac{1}{2} + Fw\left[-\frac{1-2F-2\epsilon}{2(c+w)} + \frac{c+w-2Fw-2\epsilon c-2\epsilon w}{2(c+w)^2}\right] + F\left[1 - \frac{c+w-2Fw-2\epsilon c-2\epsilon w}{2(c+w)}\right]\right) = 0,$$

which simplifies to

$$w^2 + 2cw - \frac{2bcF - c^2 - Fc^2 + 2\epsilon Fc^2}{1 + F + 2\epsilon F + 2F^2} = 0\,.$$

Solving the above quadratic equation in w, we have two solutions,

$$\frac{-2c - \sqrt{4c^2 + 4\frac{2bcF - c^2 - Fc^2 + 2\epsilon Fc^2}{1+F+2\epsilon F+2F^2}}}{2} \quad \text{and} \quad \frac{-2c + \sqrt{4c^2 + 4\frac{2bcF - c^2 - Fc^2 + 2\epsilon Fc^2}{1+F+2\epsilon F+2F^2}}}{2}\,,$$

which simplify to

$$-c - \sqrt{\frac{2cF(b + 2\epsilon c + cF)}{1 + F(1 + 2\epsilon + 2F)}} \quad \text{and} \quad -c + \sqrt{\frac{2cF(b + 2\epsilon c + cF)}{1 + F(1 + 2\epsilon + 2F)}}\,.$$

Of these, only the latter can be non-negative, which is the case as long as $b \geq c\left(\frac{1+F(1-2\epsilon)}{2F}\right)$.

The second-order condition for this optimization problem is

$$-\frac{2cF(b + 2\epsilon c + cF)}{(c + w)^3} < 0\,,$$

which holds for any (admissible) parameter values and a non-negative w.

In sum, the equilibrium reward factor w^* is

$$w^* = \begin{cases} \sqrt{\frac{2cF(b+2\epsilon c+cF)}{1+F(1+2\epsilon+2F)}} - c & \text{if } b > c\left(\frac{1+F(1-2\epsilon)}{2F}\right); \\ 0 & \text{otherwise.} \end{cases}$$

The illustration on p. 186, based on parameters $c = 1$, $F = \frac{2}{10}$, $\epsilon = \frac{1}{10}$, and $b = 70$, now yields $S^* = 0.29$, $\theta^* = 0.34$, $\phi^* = 0.78$, and $w^* = 3.62$.

The threshold θ^* is decreasing in w^* as reported in section A.4 of the supplementary appendix. But to account for the indirect effect of the parameters b, c, F, and ϵ on θ^*, its total derivatives must be based on the corrected w^*. Treating θ^* as a function of w^*, i.e. substituting the expression for w^* when it is positive into the expression for θ^*, and differentiating with respect to each of the parameters, we get

$$\begin{aligned}
\frac{d\theta^*}{db} &= -\frac{\sqrt{cF(1+F+2\epsilon F+2F^2)}}{[2(b+2\epsilon c+cF)]^{\frac{3}{2}}} < 0\,; \\
\frac{d\theta^*}{dc} &= \frac{b\sqrt{cF(1+F+2\epsilon F+2F^2)}}{c[2(b+2\epsilon c+cF)]^{\frac{3}{2}}} > 0\,; \\
\frac{d\theta^*}{d\epsilon} &= \frac{\sqrt{cF}[bF-c(1+F+F^2)]}{\sqrt{2(1+F+2\epsilon F+2F^2)(b+2\epsilon c+cF)^3}} > 0 \quad \text{for} \quad b > c\left(1+\frac{1+F^2}{F}\right)\,; \\
\frac{d\theta^*}{dF} &= -1+\sqrt{\frac{\frac{1}{2}+\frac{F}{2}+\epsilon F+F^2}{\frac{bF}{c}+2\epsilon F+F^2}}\left[1-\frac{F}{2}\left(\frac{\frac{b}{c}+2\epsilon+2F}{\frac{bF}{c}+2\epsilon F+F^2}-\frac{\frac{1}{2}+\epsilon+2F}{\frac{1}{2}+\frac{F}{2}+\epsilon F+F^2}\right)\right]\,.
\end{aligned}$$

Note that while the derivatives of θ^* with respect to b and c have the same sign as those reported in the original appendix, the derivative of θ^* with respect to ϵ is positive for large enough values of b. Meanwhile, the derivative of θ^* with respect to F is complicated.

"Fraud as Insurance" Extension The equilibrium thresholds remain the same as in Equation 1. However, as $\theta \sim U[\hat{\theta}-\sigma, \hat{\theta}+\sigma]$, the incumbent's objective function becomes

$$\begin{aligned}
& b\frac{\hat{\theta}+\sigma-\theta^*}{2\sigma} - w\left(E(\theta)+E[\phi(\theta)]F\right) \\
=& b\frac{\hat{\theta}+\sigma-\theta^*}{2\sigma} - w\left(\hat{\theta}+\frac{\hat{\theta}+\sigma-S^*}{2\sigma}F\right) \\
=& b\frac{\hat{\theta}+\sigma-\frac{1}{2}+F\frac{w}{c+w}}{2\sigma} - w\left(\hat{\theta}+\frac{\hat{\theta}+\sigma-\frac{1}{2}+F\frac{w}{c+w}+\epsilon\frac{w-c}{w+c}}{2\sigma}F\right)\,.
\end{aligned}$$

Multiplying by 2σ and dropping terms that are constant in w, the incumbent's objective is (equivalent to)

$$bF\frac{w}{c+w} - w\left(2\sigma\hat{\theta} + \left[\hat{\theta} + \sigma - \frac{1}{2} + F\frac{w}{c+w} + \epsilon\frac{w-c}{w+c}\right]F\right).$$

After the change of variables $x := c + w$, the incumbent's objective is

$$\begin{aligned}
&bF\frac{x-c}{x} - (x-c)\left(2\sigma\hat{\theta} + \left[\hat{\theta} + \sigma - \frac{1}{2} + F\frac{x-c}{x} + \epsilon\frac{x-2c}{x}\right]F\right) \\
\equiv &bF\frac{x-c}{x} - (x-c)\left(2\sigma\hat{\theta} + \left[\hat{\theta} + \sigma - \frac{1}{2}\right]F\right) - F^2\frac{(x-c)^2}{x} - F\epsilon\frac{(x-c)(x-2c)}{x}
\end{aligned}$$

which is equivalent to

$$-\frac{bFc}{x} - x\left(2\sigma\hat{\theta} + \left[\hat{\theta} + \sigma - \frac{1}{2}\right]F\right) - F^2\left(x + \frac{c^2}{x}\right) - F\epsilon\left(x + 2\frac{c^2}{x}\right).$$

The first-order condition is then

$$0 = \frac{bFc}{x^2} - \left(2\sigma\hat{\theta} + \left[\hat{\theta} + \sigma - \frac{1}{2}\right]F\right) - F^2\left(1 - \frac{c^2}{x^2}\right) - F\epsilon\left(1 - 2\frac{c^2}{x^2}\right),$$

which yields

$$x = \sqrt{\frac{bFc + F^2c^2 + 2\epsilon Fc^2}{2\sigma\hat{\theta} + \left[\hat{\theta} + \sigma - \frac{1}{2}\right]F + F^2 + F\epsilon}},$$

or

$$w^* = \sqrt{\frac{bFc + F^2c^2 + 2\epsilon Fc^2}{2\sigma\hat{\theta} + \left[\hat{\theta} + \sigma - \frac{1}{2}\right]F + F^2 + F\epsilon}} - c.$$

This is positive when

$$\frac{bFc + F^2c^2 + 2\epsilon Fc^2}{2\sigma\hat{\theta} + \left[\hat{\theta} + \sigma - \frac{1}{2}\right] F + F^2 + F\epsilon} > c^2$$

$$\iff bFc + F^2c^2 + 2\epsilon Fc^2 > c^2 \left(2\sigma\hat{\theta} + \left[\hat{\theta} + \sigma - \frac{1}{2}\right] F + F^2 + F\epsilon\right)$$

$$\iff bF + F^2c + 2\epsilon Fc > c \left(2\sigma\hat{\theta} + \left[\hat{\theta} + \sigma - \frac{1}{2}\right] F + F^2 + F\epsilon\right)$$

$$\iff bF > c \left(2\sigma\hat{\theta} + \left[\hat{\theta} + \sigma - \frac{1}{2}\right] F - F\epsilon\right)$$

$$\iff b > c \left(2\frac{\sigma\hat{\theta}}{F} + \hat{\theta} + \sigma - \frac{1}{2} - \epsilon\right).$$

We highlight that in the range of admissible σ, we have

$$\frac{\partial w^*}{\partial \sigma} < 0,$$

so that more prior uncertainty about the incumbent's popularity, *reduces* the incentives the incumbent offers for fraud.

Furthermore, in footnote 27 (p. 188) we should require $\sigma > \frac{F}{2} + 2\epsilon$ rather than $\sigma > \frac{F}{2}$, so that for appropriately chosen $\hat{\theta}$, the prior covers the interval $\left[\frac{1}{2} - F - 2\epsilon, \frac{1}{2} + 2\epsilon\right]$. This guarantees that the dominance regions cover all values of S_i for which the agent's posterior about θ is not equal to $U[S_i - \epsilon, S_i + \epsilon]$.

"Differences in Competitiveness" Extension The payoff offered to agent i should be $w(R_i - \alpha P_i)$ rather than $w(R_i - P_i)$ (p. 189), so that: (i) agents' rewards are correctly adjusted for their district's popularity, instead of agents in popular districts being systematically punished; (ii) as $\alpha \to 0$, the model converges to the baseline. The agent's payoff then becomes $w\left[(1-\alpha)S_i + \mathbb{1}_{a_i=f}F\right]$ when the incumbent wins and $-cF\mathbb{1}_{a_i=f}$ when the incumbent loses.[1]

The expressions for S^*, θ^* and ϕ^* given in p. 8 of the Online Appendix are correct. The incumbent's objective is then

$$\begin{aligned}
&b[1-\theta^*] - w\left[E(R_i) - \alpha E(P_i)\right] \\
=&b[1-\theta^*] - w\left[(1-\alpha)E(S_i) + E[\phi(\theta)]F\right] \\
=&b[1-\theta^*] - w\left[(1-\alpha)\frac{1}{2} + (1-S^*)F\right] \\
=&b\left[1-\frac{\frac{1}{2}-\frac{w}{w+c}F-\alpha\pi}{1-\alpha}\right] - w\left[\frac{1-\alpha}{2} + \left(1-\frac{\frac{1}{2}-\frac{w}{w+c}F-\alpha\pi}{1-\alpha} + \epsilon\frac{w-c}{w+c}\right)F\right]
\end{aligned}$$

Dropping terms that are constant in w, this is equivalent to

$$\frac{bF}{1-\alpha}\frac{w}{w+c} - w\left[\frac{1-\alpha}{2} + \left(1-\frac{\frac{1}{2}-\frac{w}{w+c}F-\alpha\pi}{1-\alpha} + \epsilon\frac{w-c}{w+c}\right)F\right]$$

After the change of variables $x := w+c$, this equals

$$\frac{bF}{1-\alpha}\frac{x-c}{x} - (x-c)\left[\frac{1-\alpha}{2} + \left(1-\frac{\frac{1}{2}-\frac{x-c}{x}F-\alpha\pi}{1-\alpha} + \epsilon\frac{x-2c}{x}\right)F\right]$$

which is equivalent to

$$-\frac{bF}{1-\alpha}\frac{c}{x} - x\left[\frac{1-\alpha}{2} + \left(1-\frac{\frac{1}{2}-\alpha\pi}{1-\alpha}\right)F\right] - \frac{(x-c)^2}{x}\frac{F^2}{1-\alpha} - \frac{(x-c)(x-2c)}{x}\epsilon F$$

or to

$$-\frac{bF}{1-\alpha}\frac{c}{x} - x\left[\frac{1-\alpha}{2} + \left(1-\frac{\frac{1}{2}-\alpha\pi}{1-\alpha}\right)F\right] - \left(x+\frac{c^2}{x}\right)\frac{F^2}{1-\alpha} - \left(x+\frac{2c^2}{x}\right)\epsilon F$$

[1]Due to a typo on the original payoff table (p. 7 of the Online Appendix), the original solution uses $w\left[S_i - P_i + \mathbb{1}_{a_i=f}F\right]$ as the agent's reward, which equals neither $w(R_i-\alpha P_i)$ nor $w(R_i - P_i)$.

The first-order condition is then

$$0=\frac{bF}{1-\alpha}\frac{c}{x^2}-\left[\frac{1-\alpha}{2}+\left(1-\frac{\frac{1}{2}-\alpha\pi}{1-\alpha}\right)F\right]-\left(1-\frac{c^2}{x^2}\right)\frac{F^2}{1-\alpha}-\left(1-\frac{2c^2}{x^2}\right)\epsilon F$$

which yields

$$x=\sqrt{\frac{\frac{bFc}{1-\alpha}+\frac{F^2c^2}{1-\alpha}+2c^2\epsilon F}{\frac{1-\alpha}{2}+\left(1-\frac{\frac{1}{2}-\alpha\pi}{1-\alpha}\right)F+\frac{F^2}{1-\alpha}+\epsilon F}}$$

or

$$w^*=\sqrt{\frac{\frac{bFc}{1-\alpha}+\frac{F^2c^2}{1-\alpha}+2c^2\epsilon F}{\frac{1-\alpha}{2}+\left(1-\frac{\frac{1}{2}-\alpha\pi}{1-\alpha}\right)F+\frac{F^2}{1-\alpha}+\epsilon F}}-c.$$

This is positive when

$$\begin{aligned}
&\frac{\frac{bFc}{1-\alpha}+\frac{F^2c^2}{1-\alpha}+2c^2\epsilon F}{\frac{1-\alpha}{2}+\left(1-\frac{\frac{1}{2}-\alpha\pi}{1-\alpha}\right)F+\frac{F^2}{1-\alpha}+\epsilon F}>c^2\\
\Longleftrightarrow&\frac{bFc}{1-\alpha}>c^2\left[\frac{1-\alpha}{2}+\left(1-\frac{\frac{1}{2}-\alpha\pi}{1-\alpha}\right)F+\frac{F^2}{1-\alpha}+\epsilon F\right]-\frac{F^2c^2}{1-\alpha}-2c^2\epsilon F\\
\Longleftrightarrow&\frac{bFc}{1-\alpha}>c^2\left[\frac{1-\alpha}{2}+\left(1-\frac{\frac{1}{2}-\alpha\pi}{1-\alpha}\right)F-\epsilon F\right]\\
\Longleftrightarrow&b>c\left[\frac{(1-\alpha)^2}{2F}+\left(1-\alpha-\frac{1}{2}+\alpha\pi\right)-\epsilon(1-\alpha)\right].
\end{aligned}$$

Paralleling our last remark in the previous section, in Eq. A.2 (p. 8 of the Online Appendix) we should require $F+\alpha\pi+(1-\alpha)2\epsilon\leq\frac{1}{2}$ and $\alpha(1-\pi-2\epsilon)\leq\frac{1}{2}-2\epsilon$ to guarantee that the dominance regions cover all values of S_i for which the agent's posterior about θ is not $U[S_i-\epsilon,S_i+\epsilon]$.

References

Rundlett, Ashlea and Milan W. Svolik. 2016. "Deliver the Vote! Micromotives and Macrobehavior in Electoral Fraud." *American Political Science Review* 110(1):180–19.